Optimización de una propagación orbital masiva: Aplicación sobre el análisis dinámico de orbitadores (Troyanos y Griegos)

II Workshop de Investigación del Centro ESenCIA
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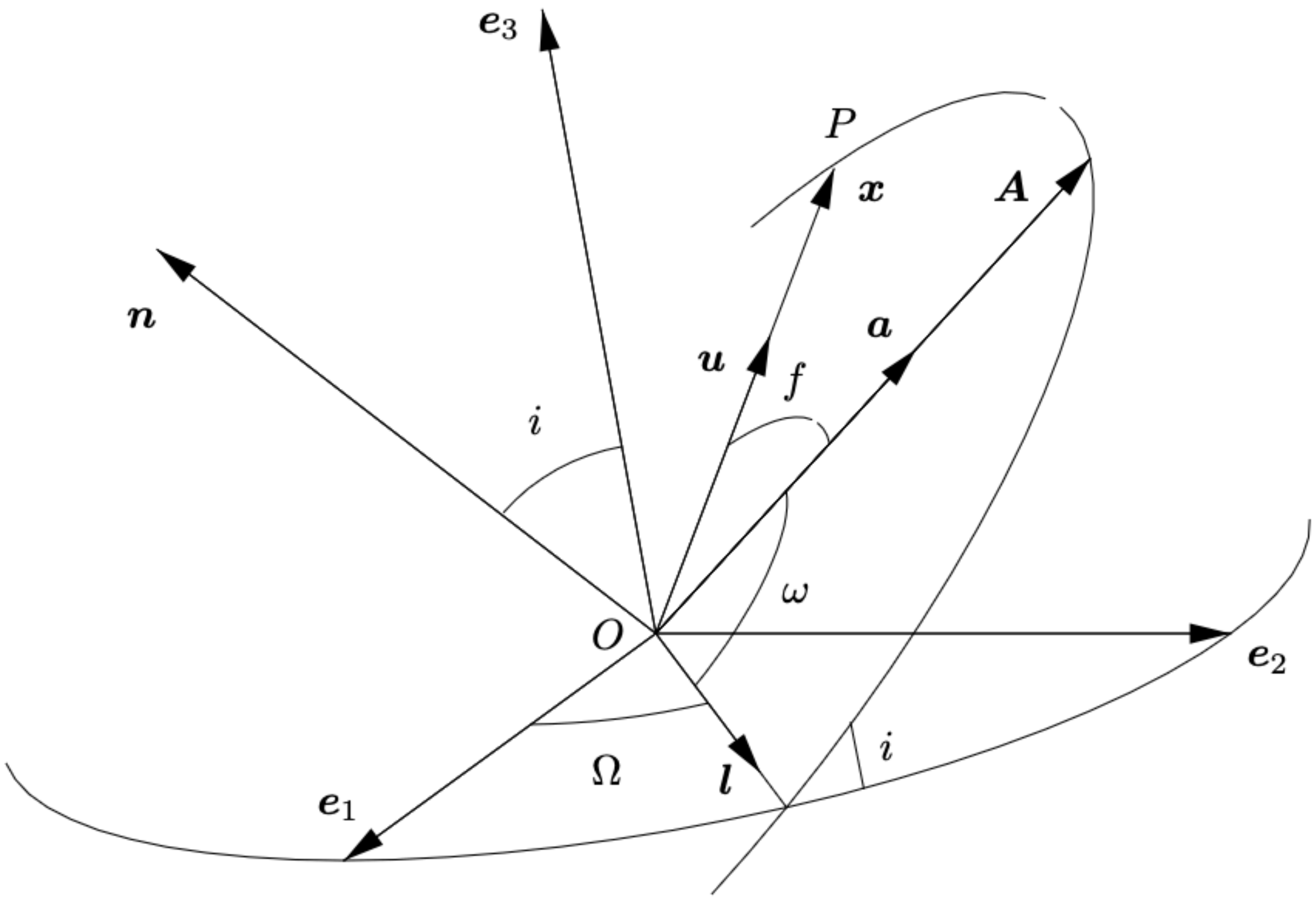
Lacruz, E.[1,2] & Casanova, D.[3,4]

(1) Centro de Física del Cosmo de Aragón - CEFCA. Plaza San Juan. 44001, Teruel. Spain.
(2) Valencian International University -VIU, C/Pintor Sorolla 21, 46002, Valecia, Spain.
(3) Centro Universitario de la Defensa- AGM, Ctra. Huesca s/n, 50090, Zaragoza, Spain.
(4) APEDIF- IUMA- University of Zaragoza, C/ Pedro Cerbuna 12, 50009, Zaragoza. Spain.

Dartnell, L., 2023

The scenario: Sun-Jupiter-Asteroid + configuration



* Easy model

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{X}(t), \quad \dot{\boldsymbol{X}}(t) = -\frac{\mu}{r^3}\,\boldsymbol{x}(t) + \mathcal{P}(t),$$

* Solution

$$\sigma_{oe} = \{a,\, e,\, i,\, \Omega,\, \omega,\, \tau\},$$

* Solution (step integration)

$$\sigma_{oe}(t) \;=\; \{a(t),\, e(t),\, i(t),\, \Omega(t),\, \omega(t),\, \tau(t)\}$$

The scenario: Sun-Jupiter-Asteroid + configuration

* Example: a propagation over 10 years by second.

  * One object (Asteroid, Space Debris, other)

Size

$$\sigma_{\mathcal{O}_k} = \cup_{i=0}^{t} \sigma_{oe}(i)$$

$$card(\sigma_{\mathcal{O}_k}) = \|\sigma_{oe}\| \times N_p$$

One billion

$$\sigma_{\mathcal{O}_k} = \cup_{i=0}^{3.1536 \times 10^8} \sigma_{oe}(i)$$

$$card(\sigma_{\mathcal{O}_k}) = 1.89216 \times 10^9$$

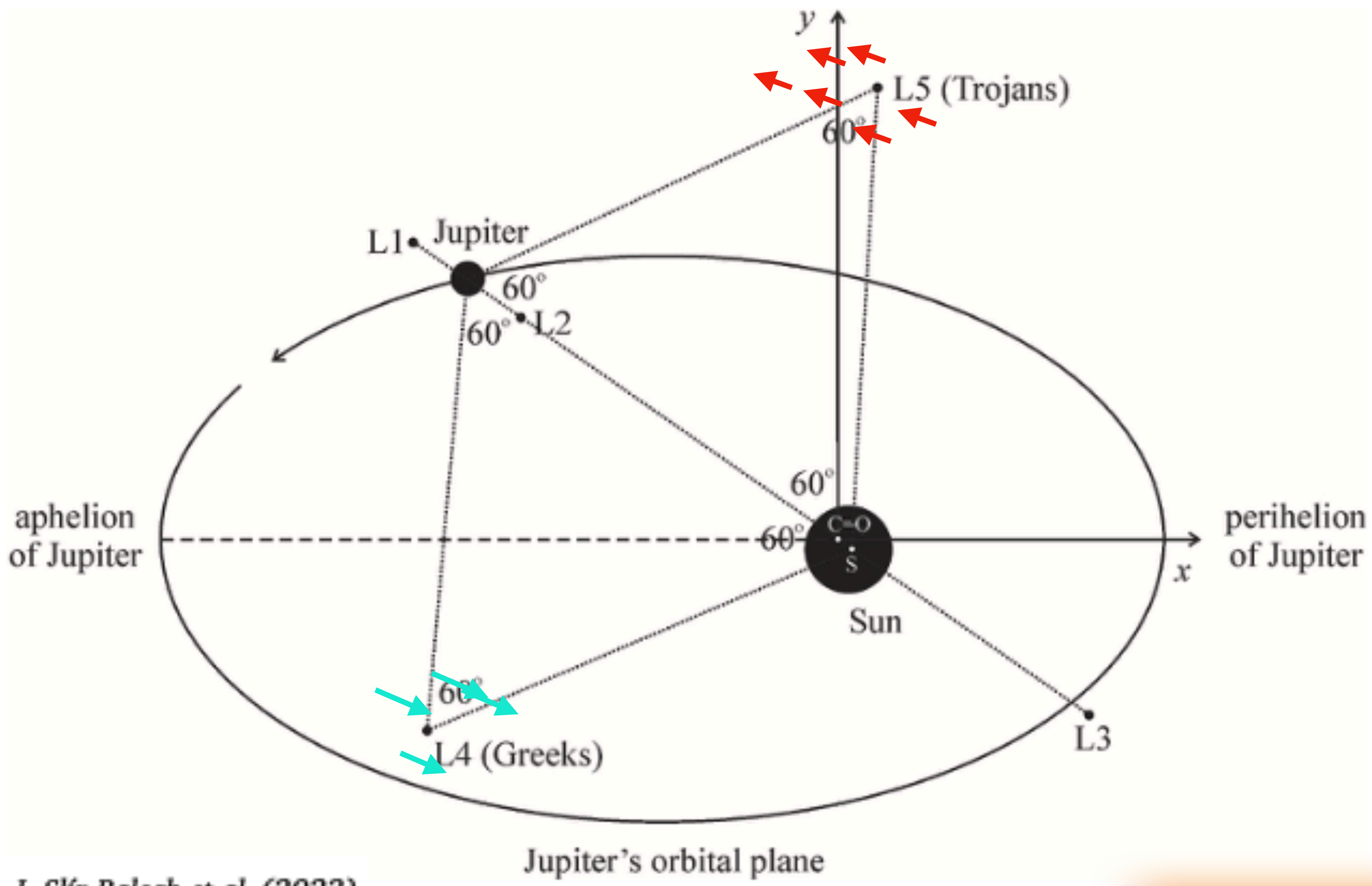
época	N_p	$card(\sigma_{\mathcal{O}_k})$
1	3.1536×10^7	1.89216×10^8
2	6.3072×10^7	3.78432×10^8
5	1.5768×10^8	9.4608×10^9
8	2.52288×10^8	1.513728×10^9
12	3.78432×10^8	2.270592×10^9
15	4.7304×10^8	2.83824×10^9
20	6.3072×10^8	3.78432×10^9
70	2.20752×10^9	1.34512×10^{10}

The scenario: Sun-Jupiter-Asteroid + configuration

*Classification of the variables and their associated parameters for the system of differential equations defined

Type	Sets	Variables
$\mathcal{K}epler$ $\mathcal{D}elaunay$	$\sigma_{oe(\mathcal{K})}$ $\sigma_{oe(\mathcal{D})}$	$a,\ e,\ i,\ \Omega,\ \omega,\ \tau$ $M,\ g,\ \Omega,\ L_d,\ G,\ H_d$
$\mathcal{W}hittaker$	$\sigma_{oe(\mathcal{W})}$	$r,\ \theta,\ \nu,\ R,\ \Theta,\ N$
$\mathcal{P}oincar\acute{e}$	$\sigma_{oe(\mathcal{P})}$	$\lambda_M,\ g_p,\ h_p,\ L_p,\ G_p,\ H_p$
$\mathcal{M}ilankovitch$	$\sigma_{oe(\mathcal{M})}$	$\boldsymbol{H},\ \boldsymbol{e},\ L$
$\mathcal{E}quinoctial$	$\sigma_{oe(\mathcal{E})}$	$n,\ a_f,\ a_g,\ L,\ \chi,\ \psi$
TLE	$\sigma_{oe(\mathcal{TLE})}$	$\bar{n}_{tk},\ e_{tk},\ i_{tk},\ \Omega_{tk},\ \omega_{tk},\ M_{tk}$

The scenario: Sun-Jupiter-Asteroid configuration



J. Slíz-Balogh et al. (2023)

<u>Orbital Dynamical Models</u>

Orbital Keplerian model : **Sun-Jupiter**

$$\dot{\boldsymbol{x}} = \boldsymbol{X}, \quad \dot{\boldsymbol{X}} = -\frac{\mu}{r^3}\boldsymbol{x},$$

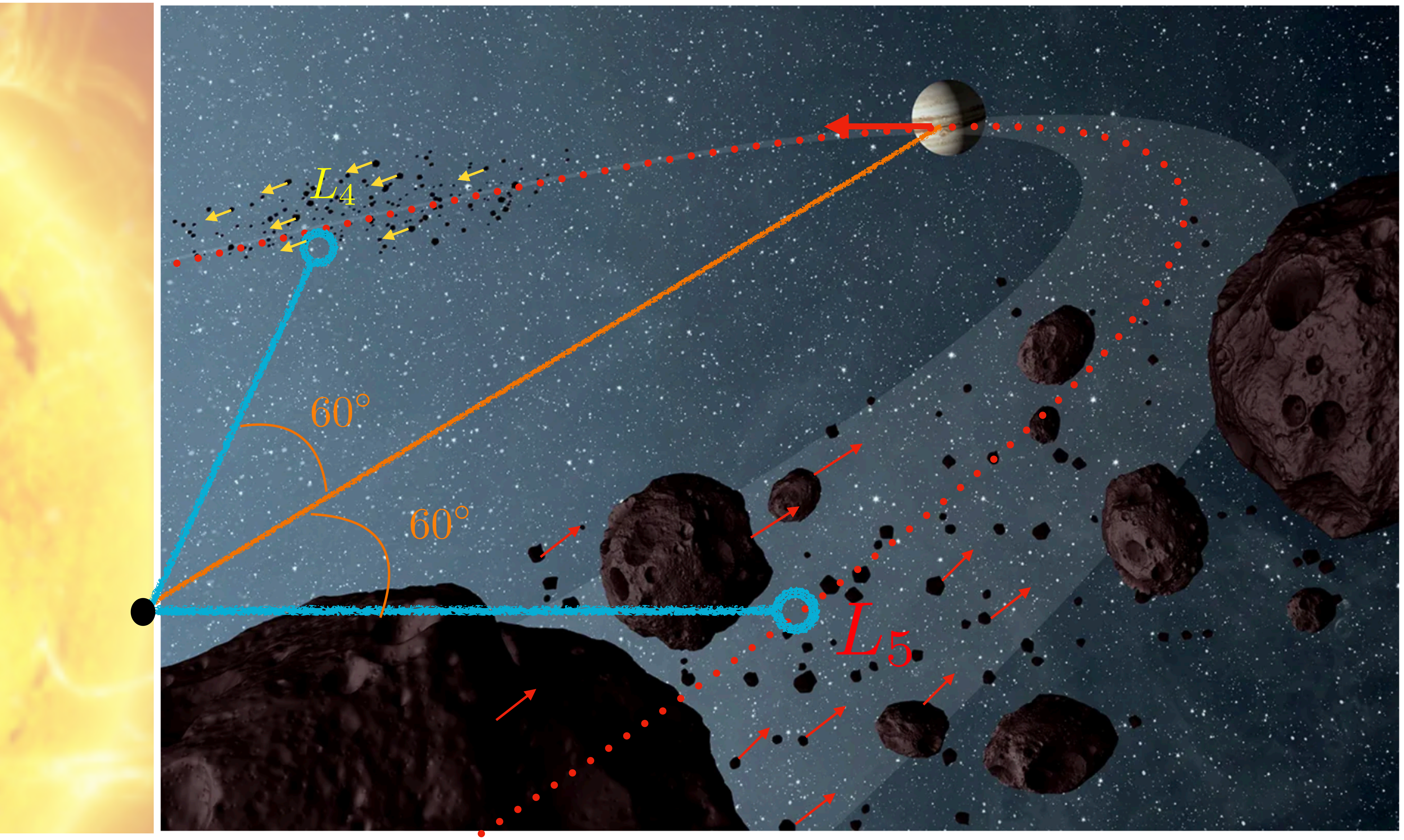
Orbital Perturbed model

Sun-Jupiter-Asteroid.

$$\ddot{\boldsymbol{x}} + \mu\frac{\boldsymbol{x}}{r^3} = \boldsymbol{\mathcal{P}},$$

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{X}, \\ \dot{\boldsymbol{X}} &= -\frac{\mu}{r^3}\boldsymbol{x} + \boldsymbol{\mathcal{P}}. \end{aligned}$$

The scenario: Sun-Jupiter-Asteroid configuration



In L_5 there are 9744 objects cataloged until 2025

✳Jupiter Trojan population.

In L_4 there are 5561 objects cataloged until 2025

✳Jupiter Greek population.

✳ Except for Pratroclus (Greek), who is at L_5, and Achilles (Trojan) is at L_4.

Trojan and Greek Camp Regions of Jupiter

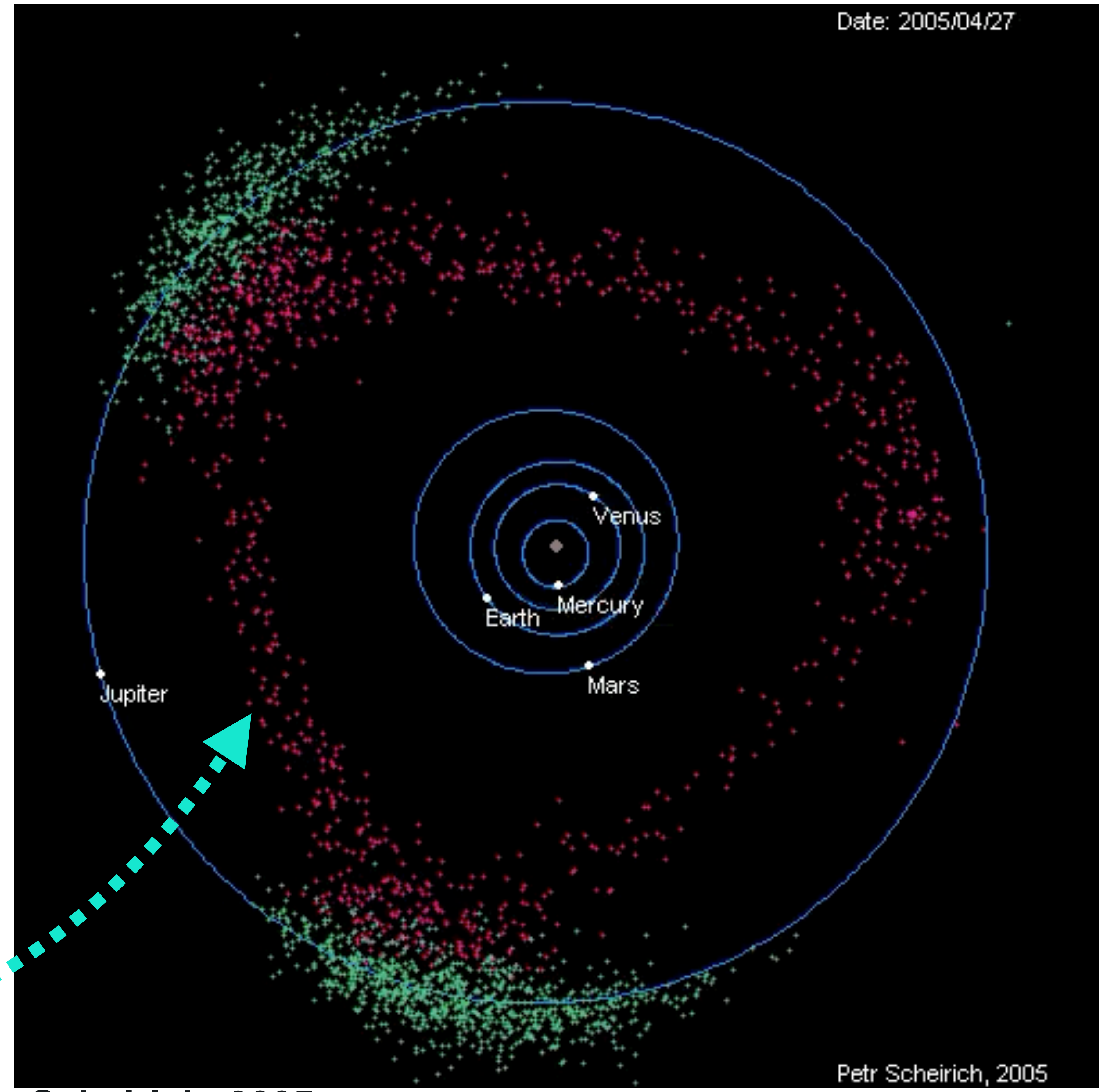
Take into account the Sun-Jupiter-Asteroid problem.

There are 15311 objects cataloged as Jovians Trojans until 2025.

Jupiter Trojans are populations that orbit around the L4 and L5 Lagrange points of Jupiter, which are located 60° in front of and behind Jupiter, respectively.

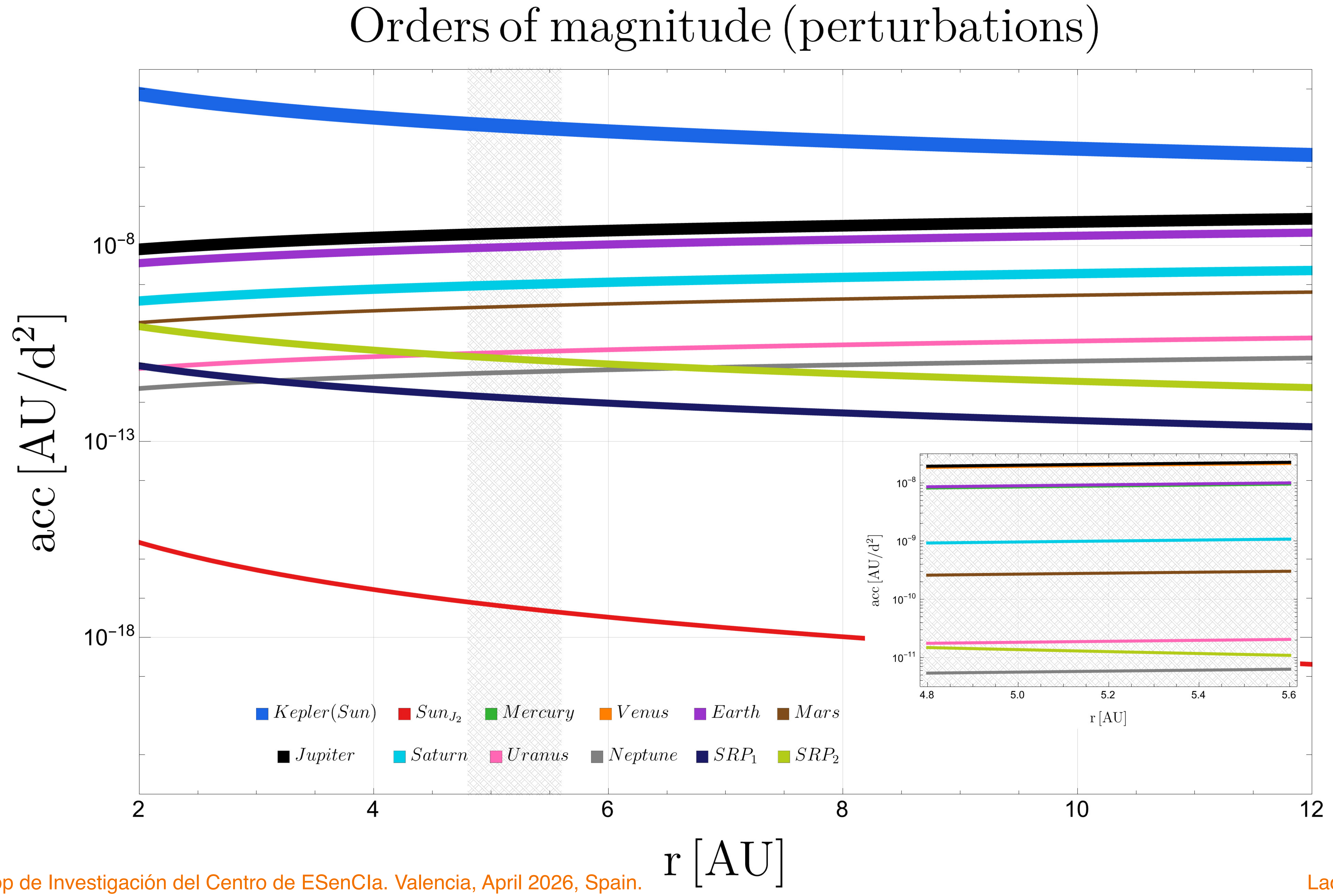
They are contained within Jupiter's 1:1 mean motion resonance, with Jupiter having a semimajor axis of 5.2 au

Jorba, Nicolás & Rodríguez, (2024).
A dynamical study of Hilda asteroids in the Circular and Elliptic RTBP



Scheirich, 2005

The scenario: Sun-Jupiter-Asteroid configuration



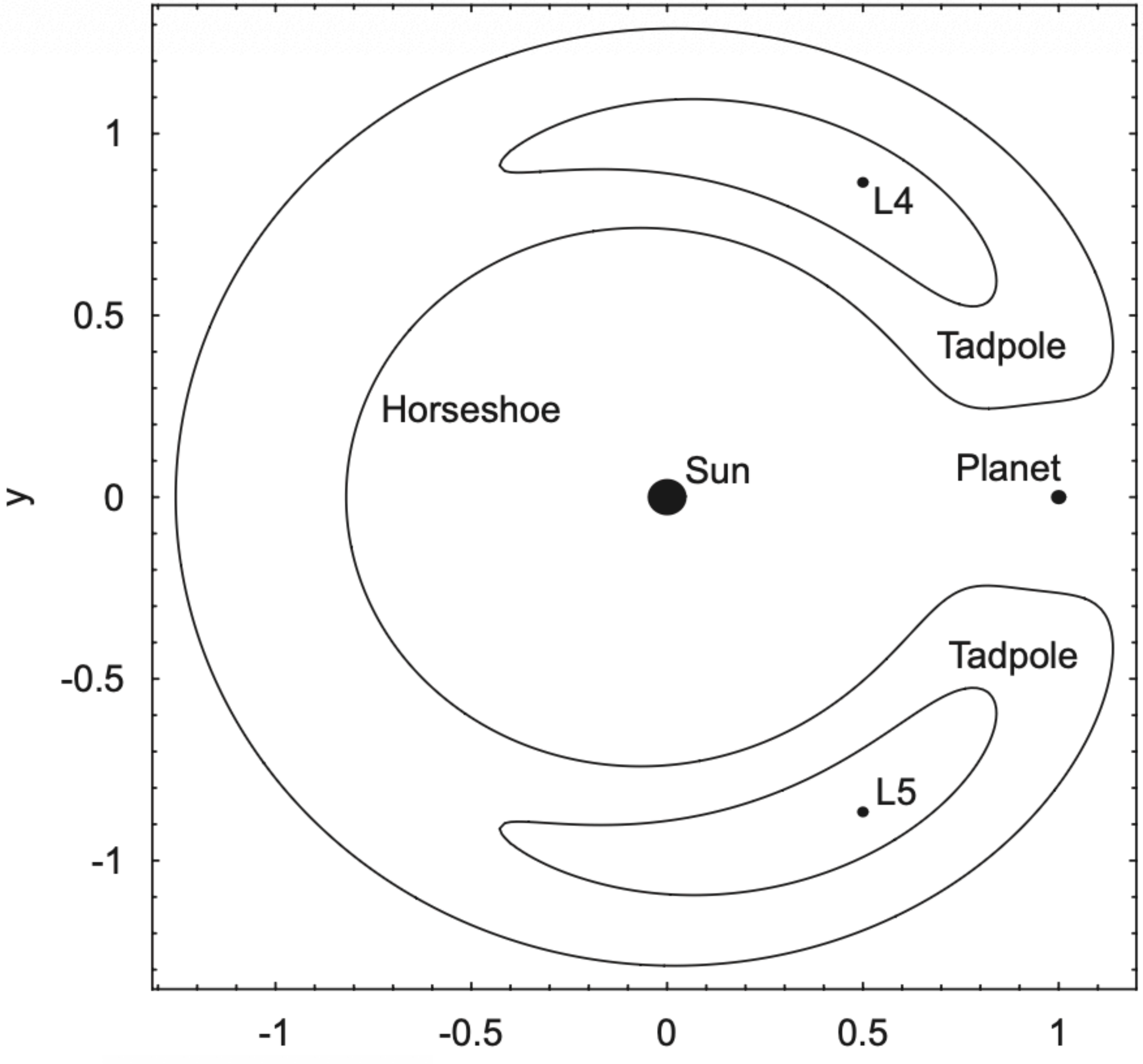
The scenario: Sun-Jupiter-Asteroid + configuration

Which orbital dynamical models are being considered?

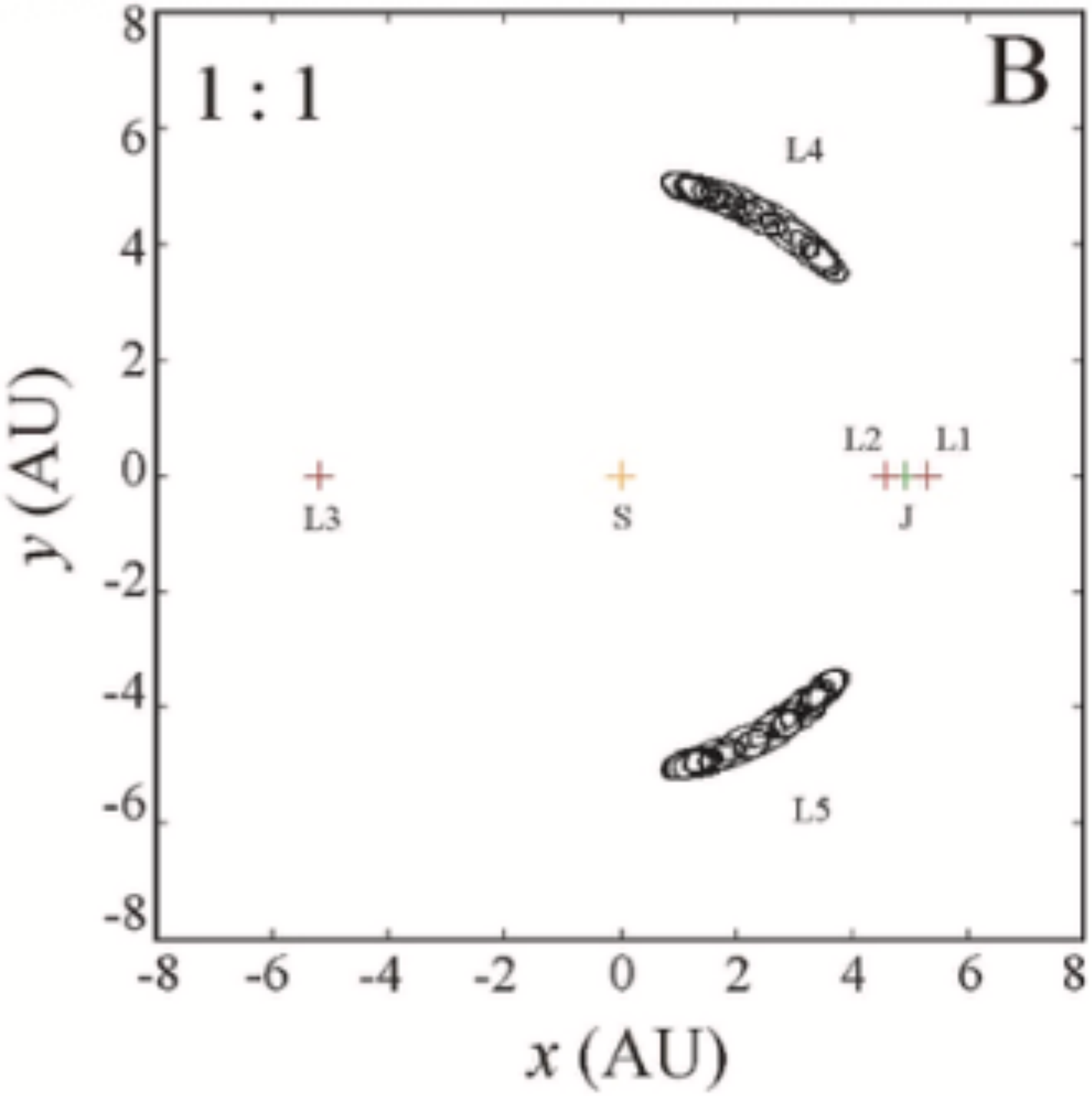
$$\ddot{\mathbf{r}} = \mathbf{a}_{\mathrm{Sun}} + \mathbf{a}_{J_2} + \mathbf{a}_{\mathrm{p}}, \qquad p = \{\mathrm{Jupiter}, \dot{\mathrm{S}}\mathrm{aturn}, \mathrm{Uranus}, \mathrm{Neptune}\}$$

$$\ddot{\mathbf{r}} = -\frac{\mu_{\mathrm{Sun}}\,\mathbf{r}}{r^3} + \sum_p \mu_p \left[\frac{\mathbf{r}_p - \mathbf{r}}{|\mathbf{r}_p - \mathbf{r}|^3} - \frac{\mathbf{r}_p}{r_p^3}\right] + \nabla\left[\frac{\mu_{\mathrm{Sun}}\, J_2\, R_{\mathrm{Sun}}^2}{2\, r^3}\left(\frac{3z^2}{r^2} - 1\right)\right]$$

The scenario: Jovian Asteroid trajectories

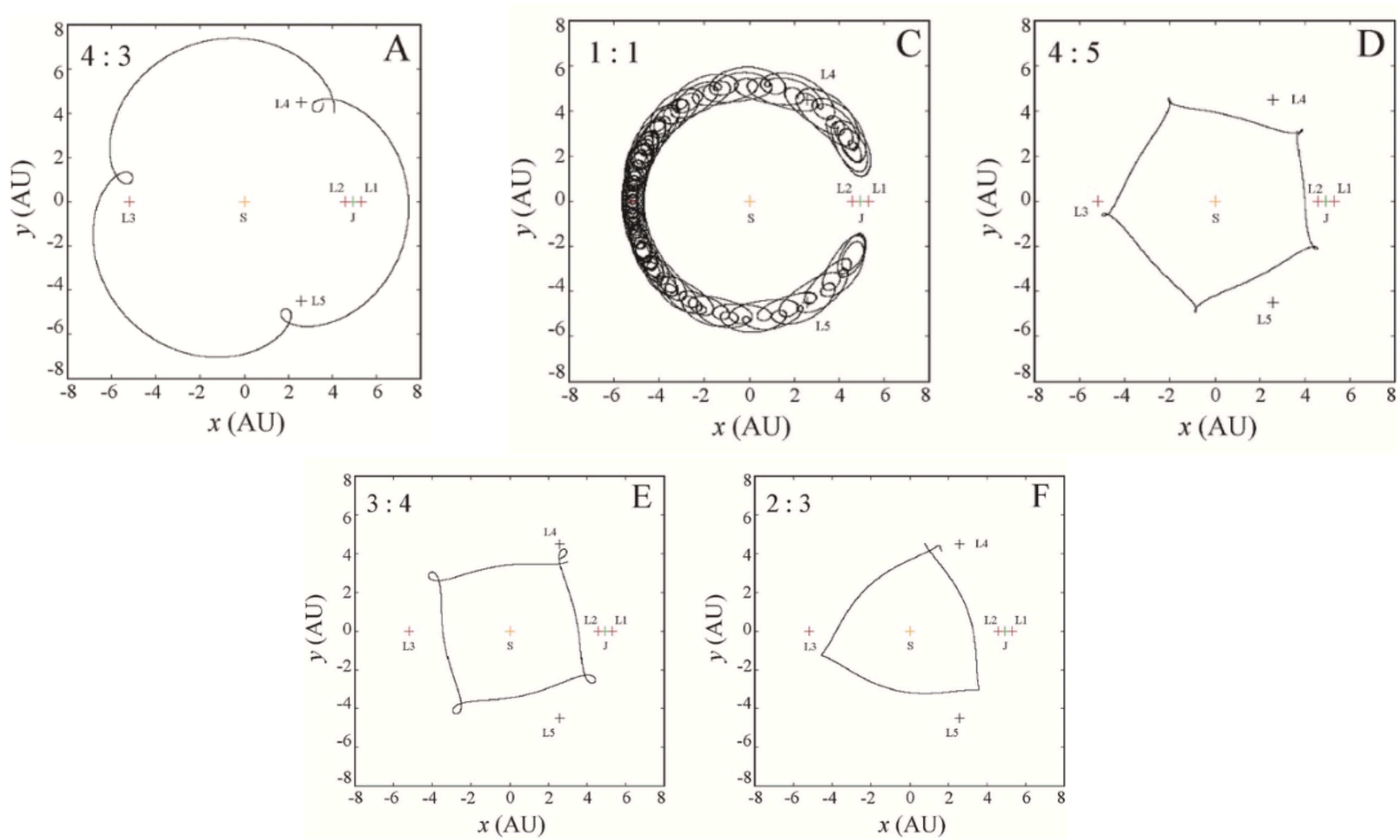


R.G. Stacey, M. Connors (2008)



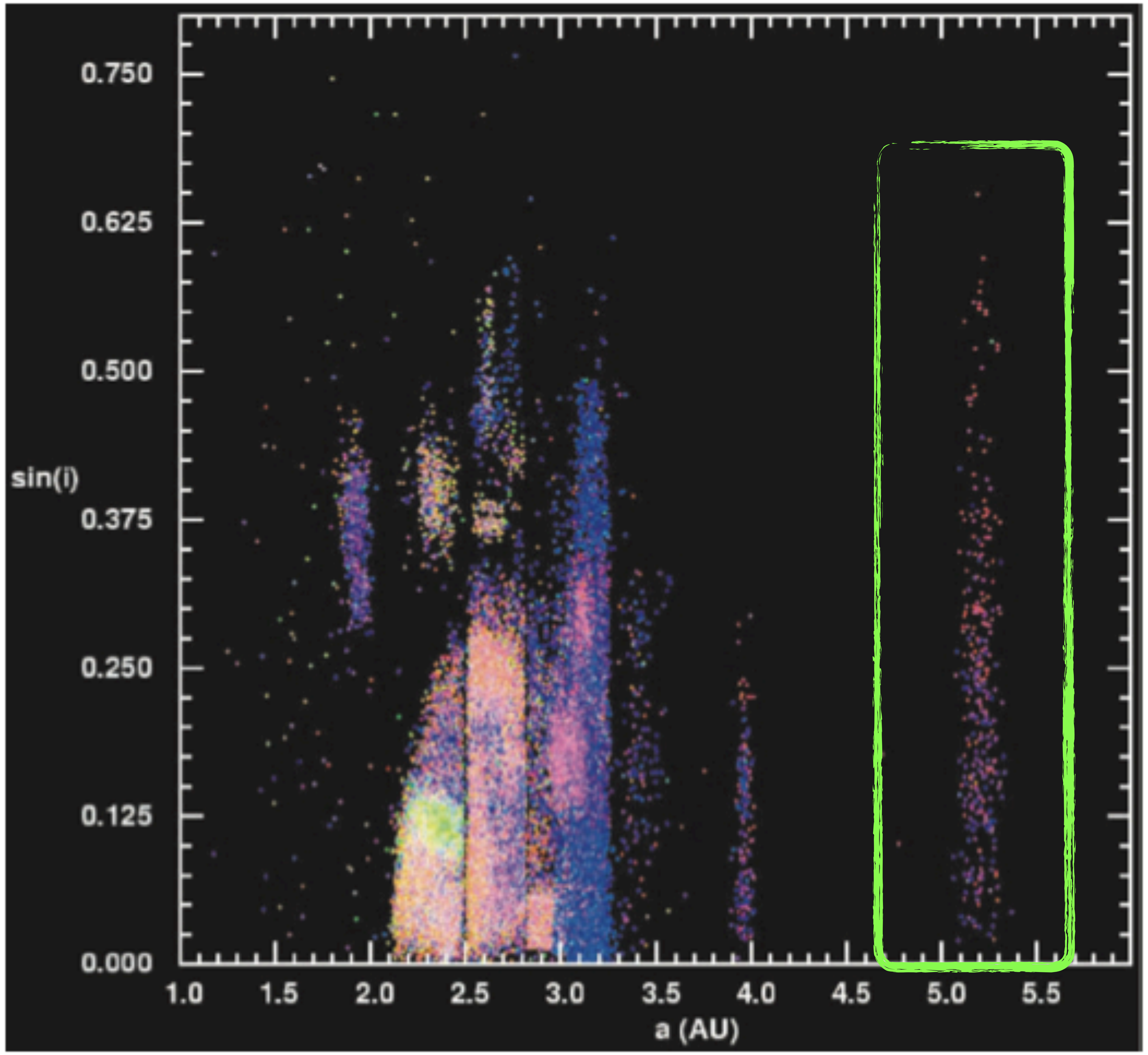
The scenario: Jovian Asteroid trajectories

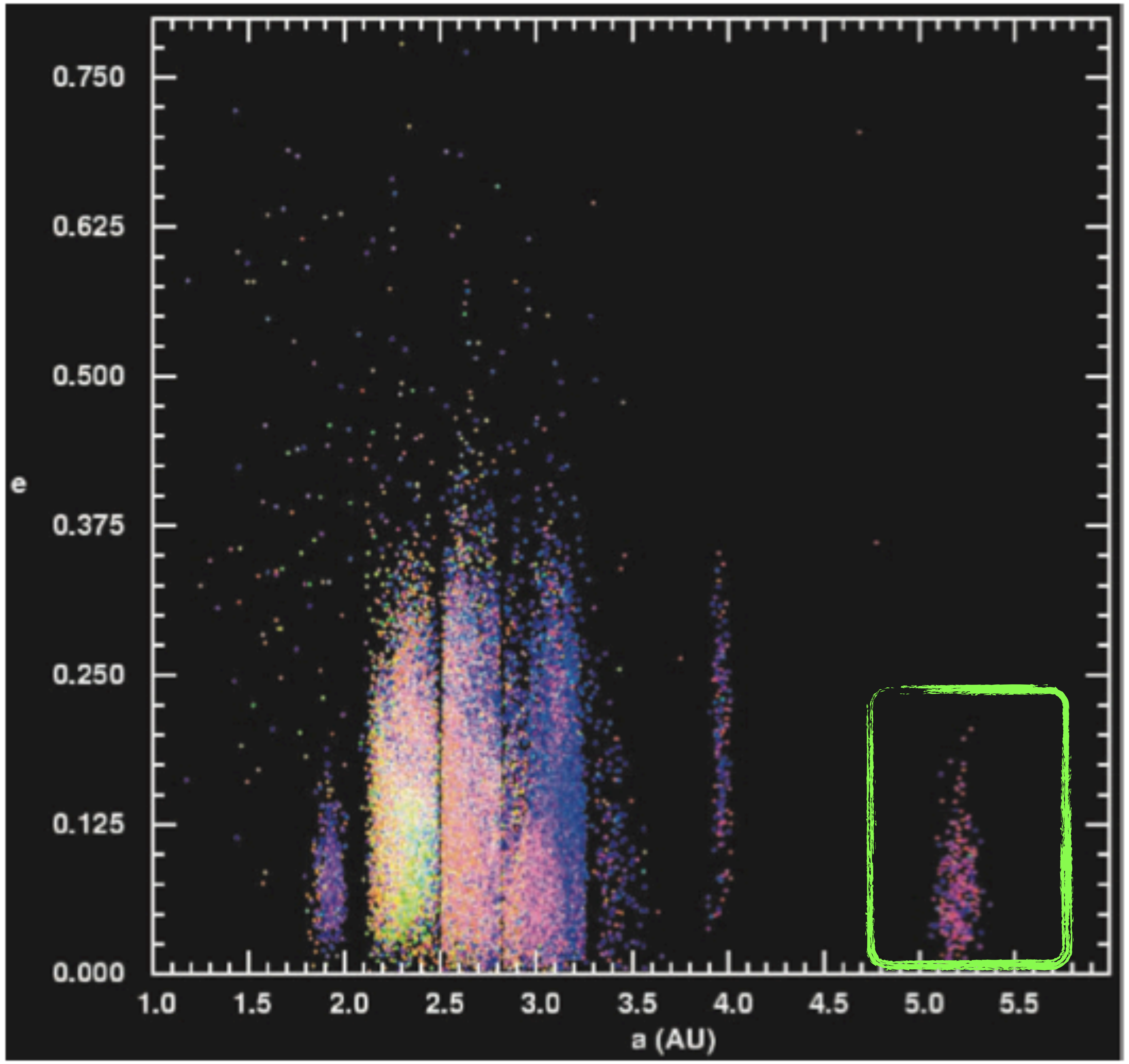
J. Slíz-Balogh et al. (2023)



The scenario: Sun-Jupiter-Asteroid + configuration

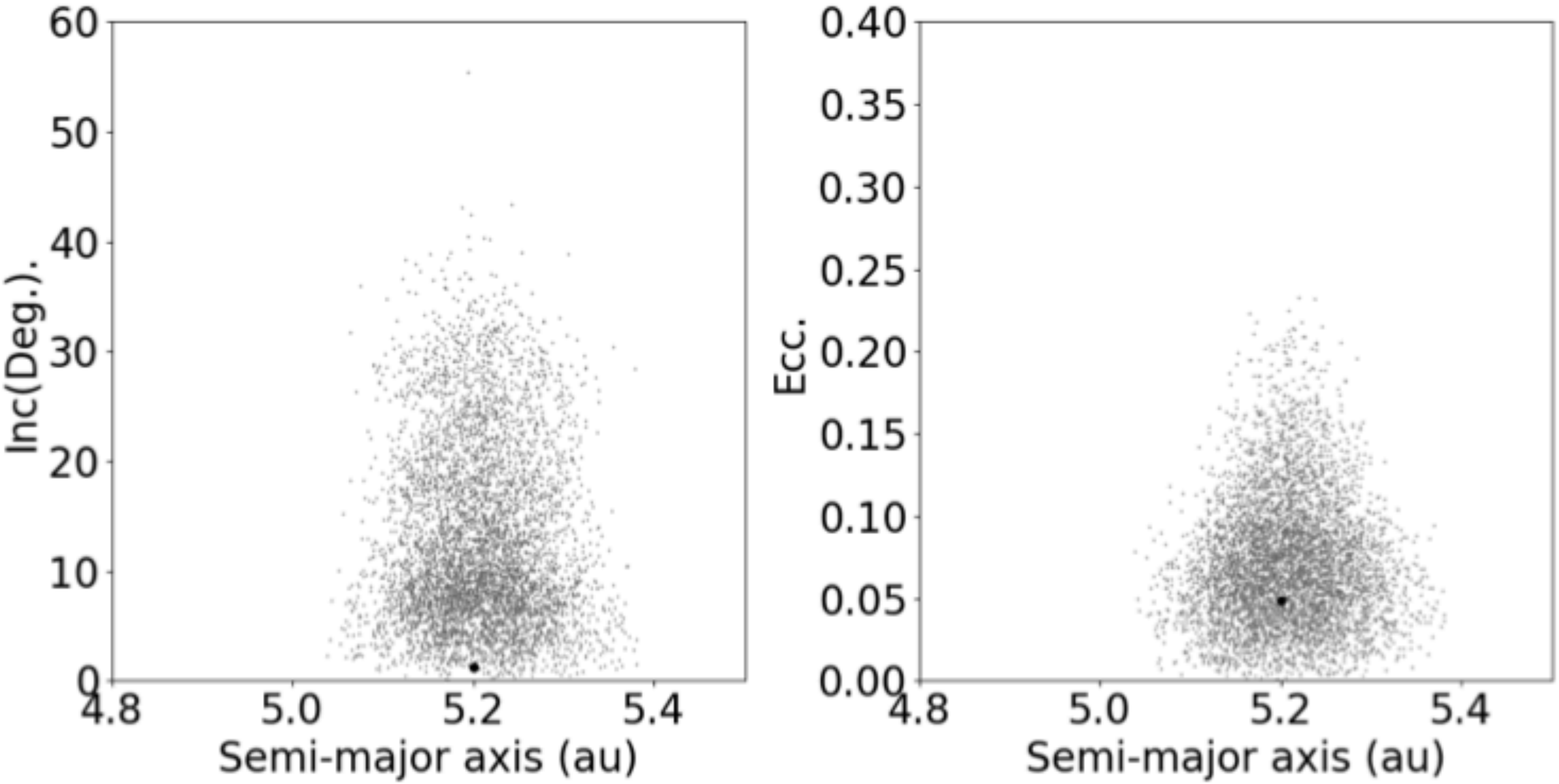
G. M. Szabó et al.(2007)



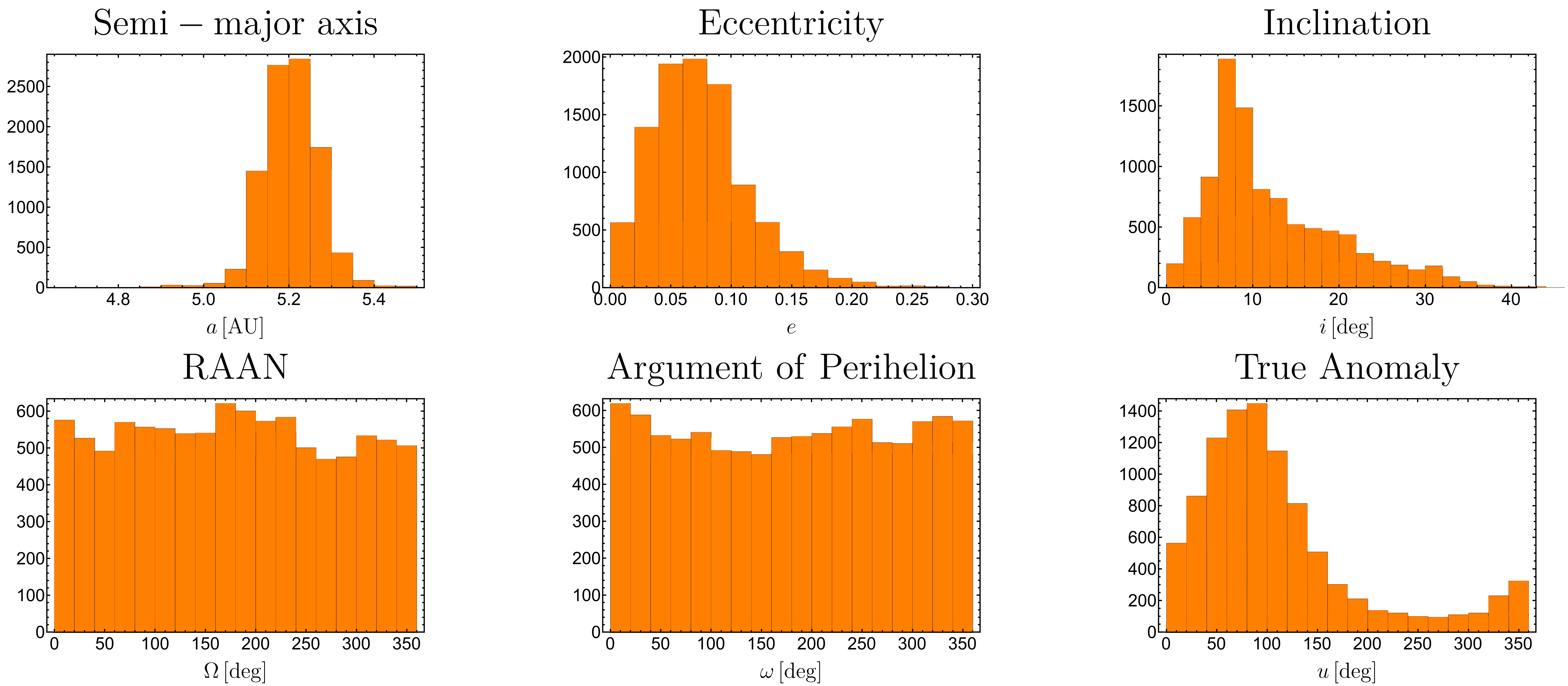


The scenario: Sun-Jupiter-Asteroid + configuration

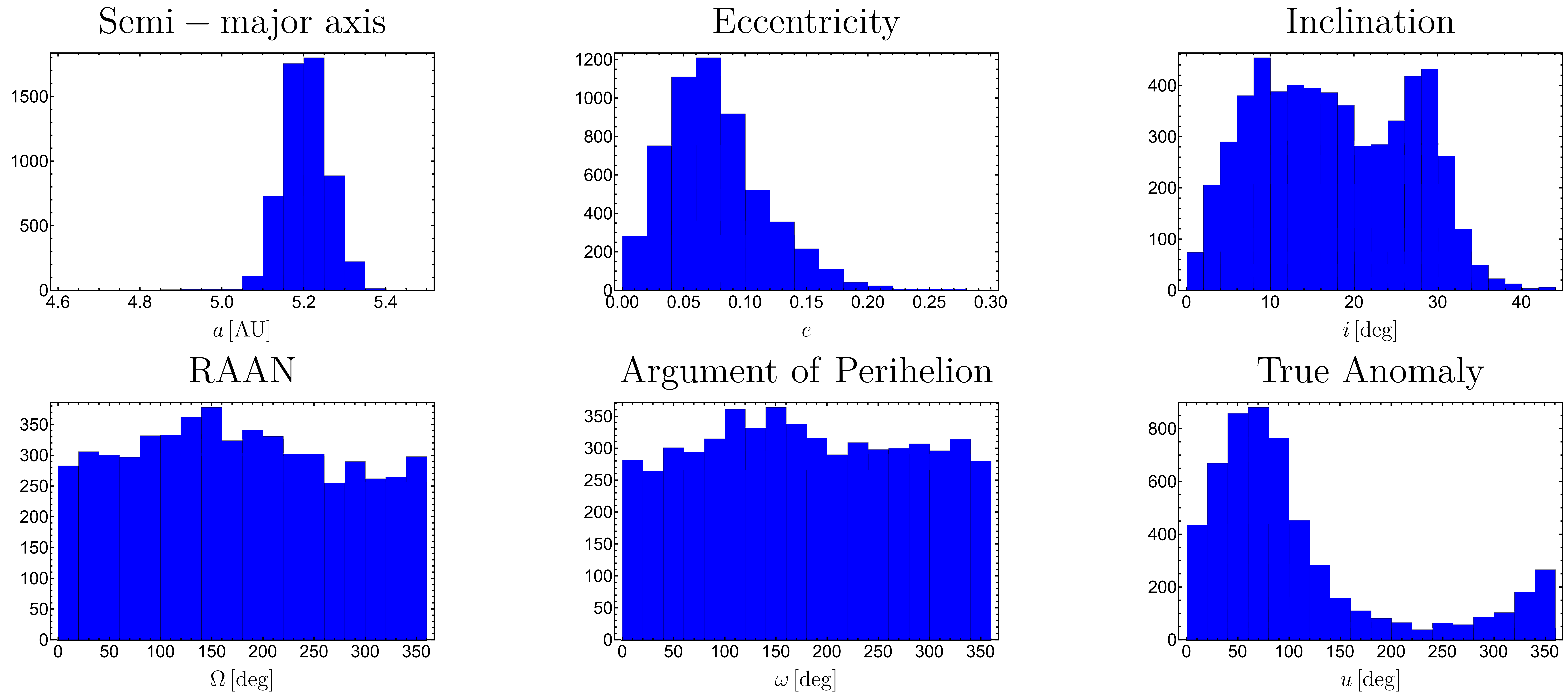
T. R. Holt et al. (2019)



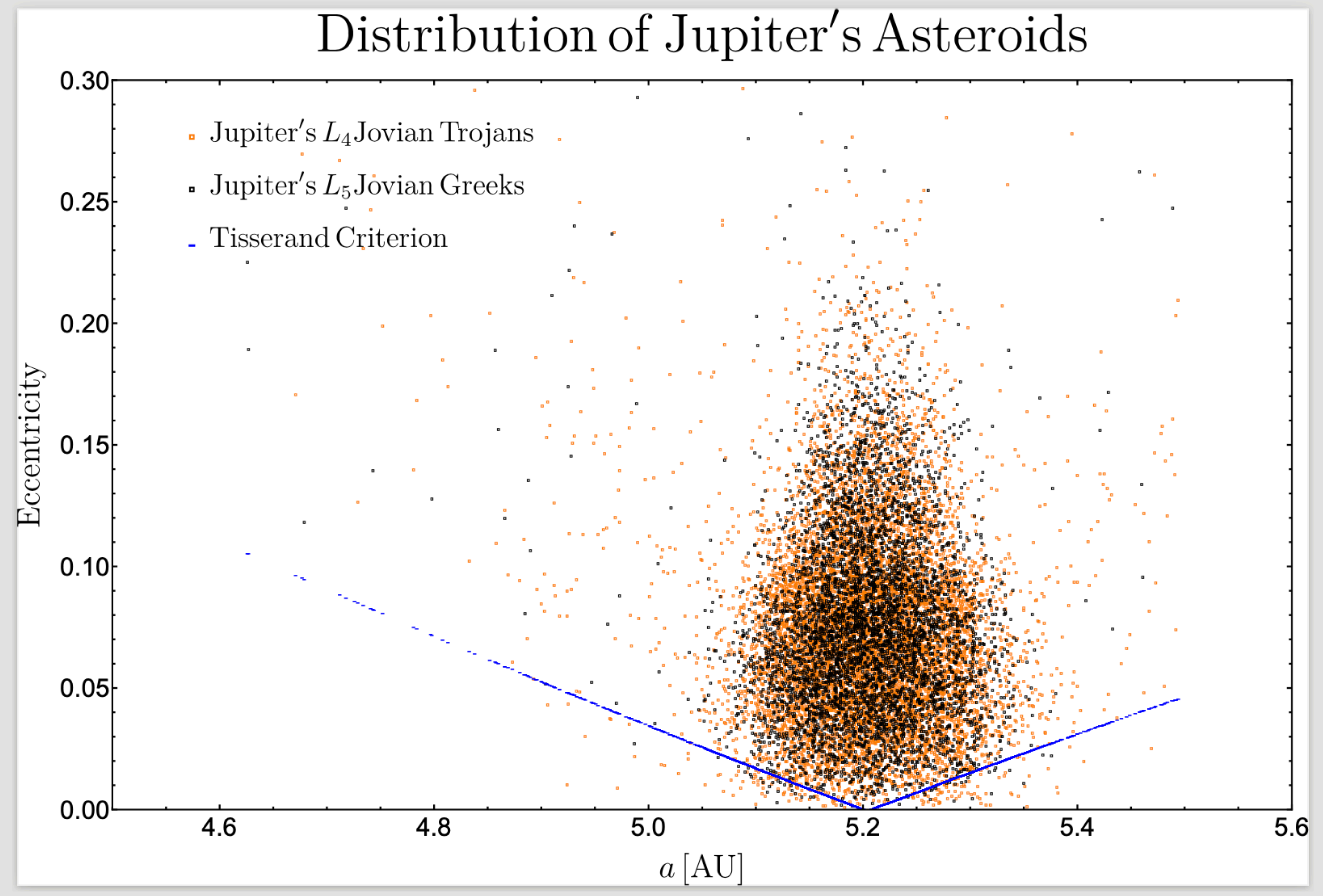
Distribution of Orbital Elements of the Trojan Asteroids



Distribution of Orbital Elements of the Greek Asteroids



Distribution Eccentricity vs Semi-major axis



Tisserand Criterion

$$T_J = \frac{a_J}{a} + 2\sqrt{\frac{a}{a_J}(1-e^2)}\cos i,$$

When $T_J = 3,$

$$e = \sqrt{1 - \left(\frac{3 - \frac{a_J}{a}}{2}\right)^2 \frac{a_J}{a}}.$$

This equation defines a locus in the (a, e) phase space.

Aim and Methodology

Determine leaders and associated objects: Trojan and Greek camps

1.- Data (https://ssd.jpl.nasa.gov/planets/orbits.html)

1.1 Jupiter Trojan Clouds (Tojan Camp and Greek Camp)

2.- Propagation:

2.1. Orbital model that represents the orbital dynamics as accurately as possible.

2.2. Propagator (Tool) named *OrbitSAT* (Abad & Lacruz, (2013)) to obtain discretized perturbed orbits.

2.3. Solver the Kepler equation (Calvo, et.al (2026)) -> Padé and Hermite approximations

3.- Algorithms:

3.1. Computation of the maximun of the minimun distances between each pair of dicretized orbits (Lacruz & Casanova, (2024), Lacruz & Casanova, (2026)).

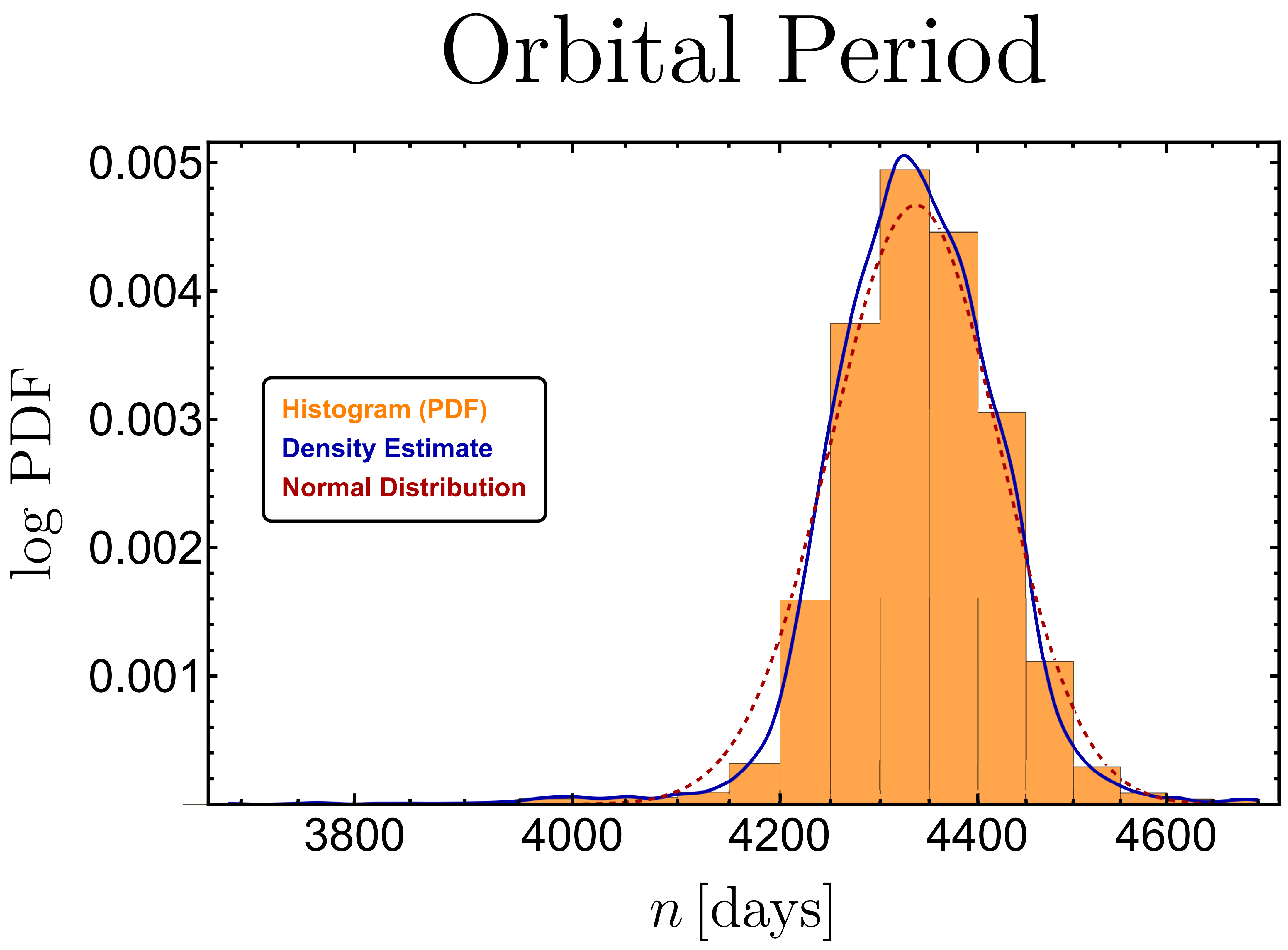
4.- Metric:

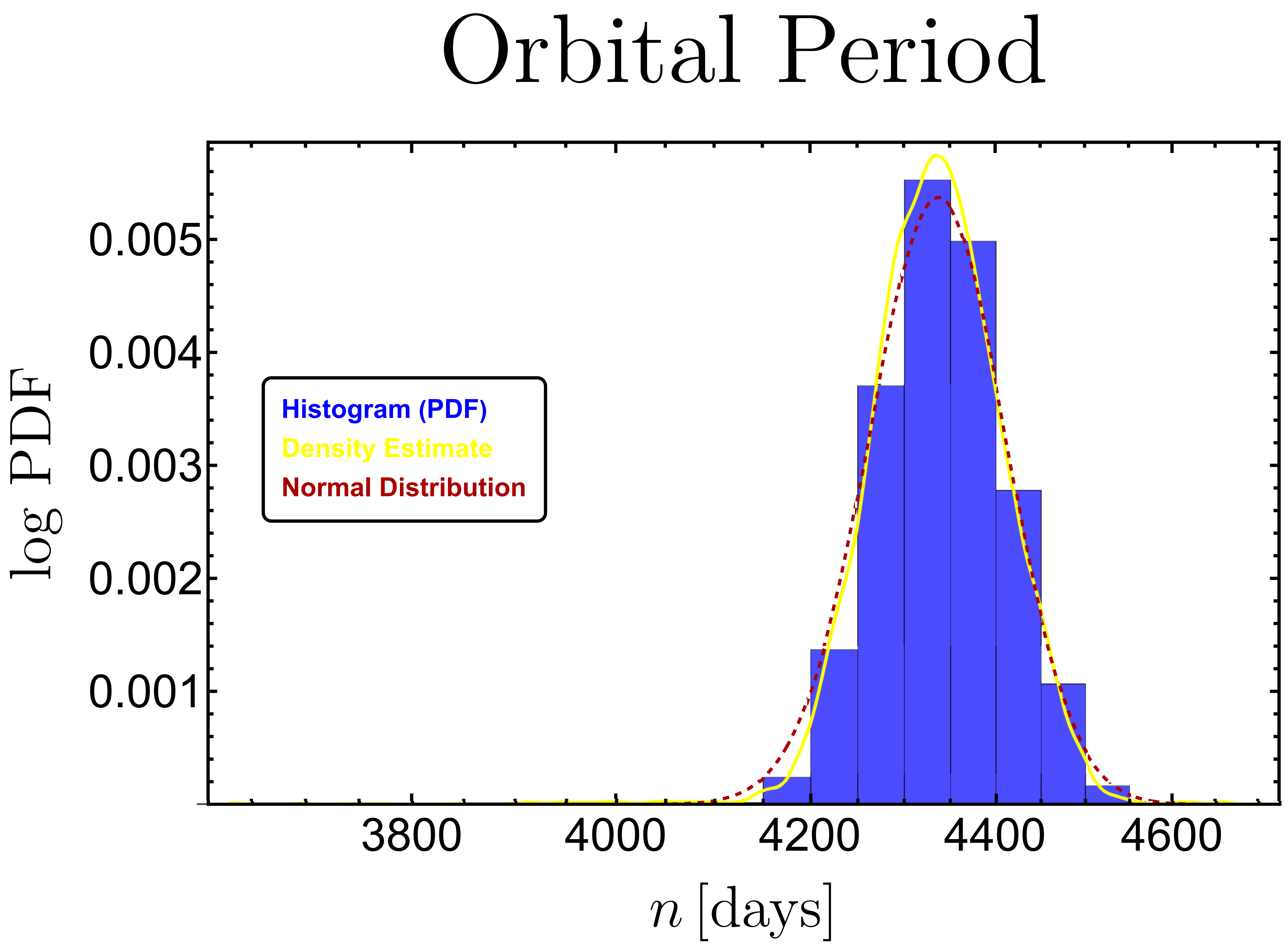
4.1. Determine a threshold distance which will allow us to identify the potential leaders and the pieces associated with them. Statistical Spatial Distribution.

Propagation of Orbtis

2.3. Use the JPL ephemerides (this is incorporated into *OrbitSAT*)

2.4. Time-Discretized Orbits with daily step.





Propagation of Orbtis

Our first situation.

Computational

Time-Cost-Efficiency

Time- Complexity

Time-Discretized Orbits with daily step (OrbitSAT)

SEQUENTIAL FORM

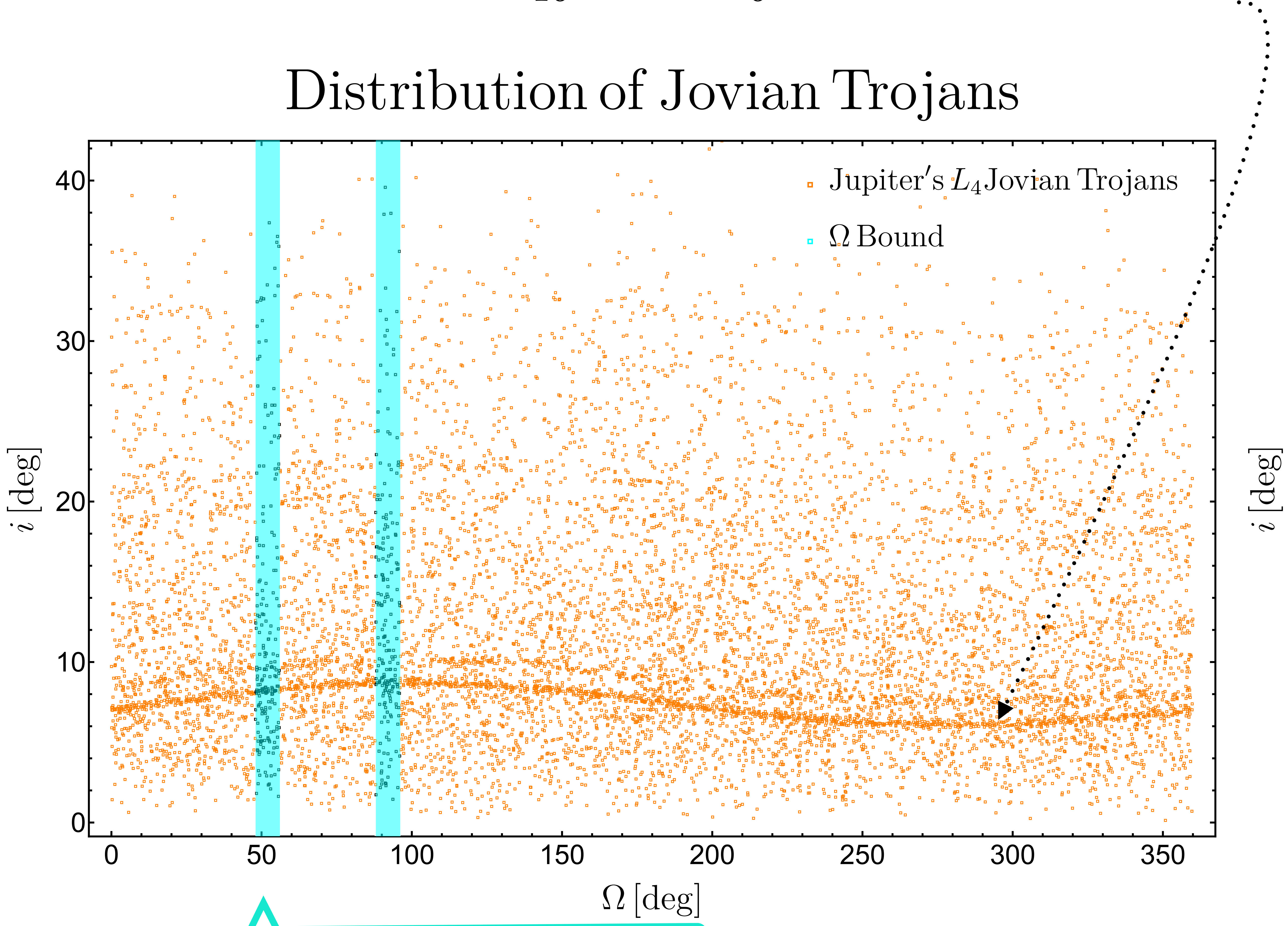
Objects	Ephemerides (Rows)	Position (3), Velocity (3), Orbital Period (1), Time (1).	Time-CPU
15311	65.837.300	4.608.861.100	54mm 46 ss

PARALLEL FORM (OpenMP)

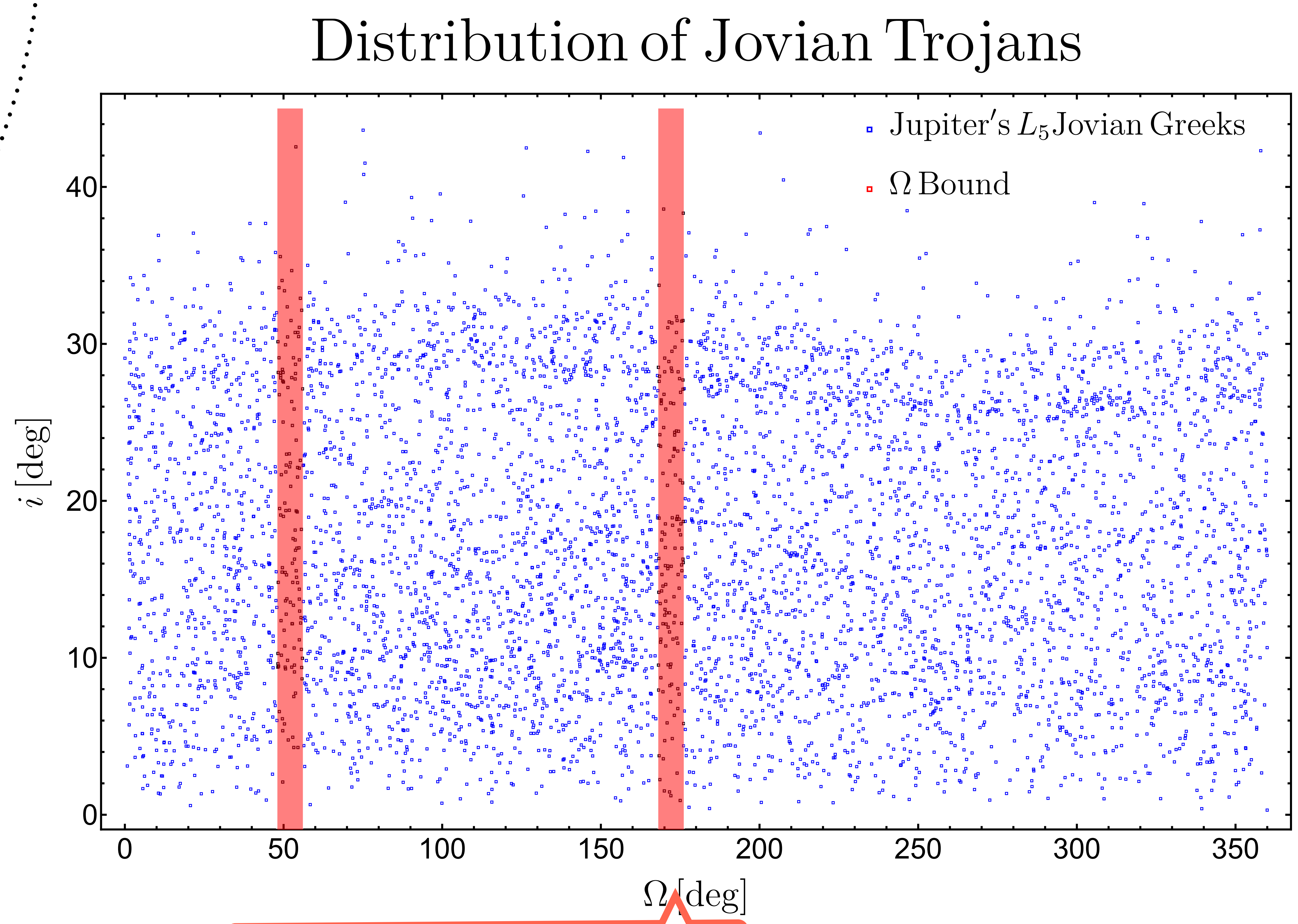
Objects	Ephemerides (Rows)	Position (3), Velocity (3), Orbital Period (1), Time (1).	Time-CPU.	Kernel-CPU
15311	65.837.300	4. 608.861.100	24mm 36ss	4 Efficiency 4 Performance Mac AM2

Distribution Inclination vs RAAN

There are a dependency between both orbital elements: The secular resonance $\nu_{16} = \dot{\Omega} - s_6 = 0.$



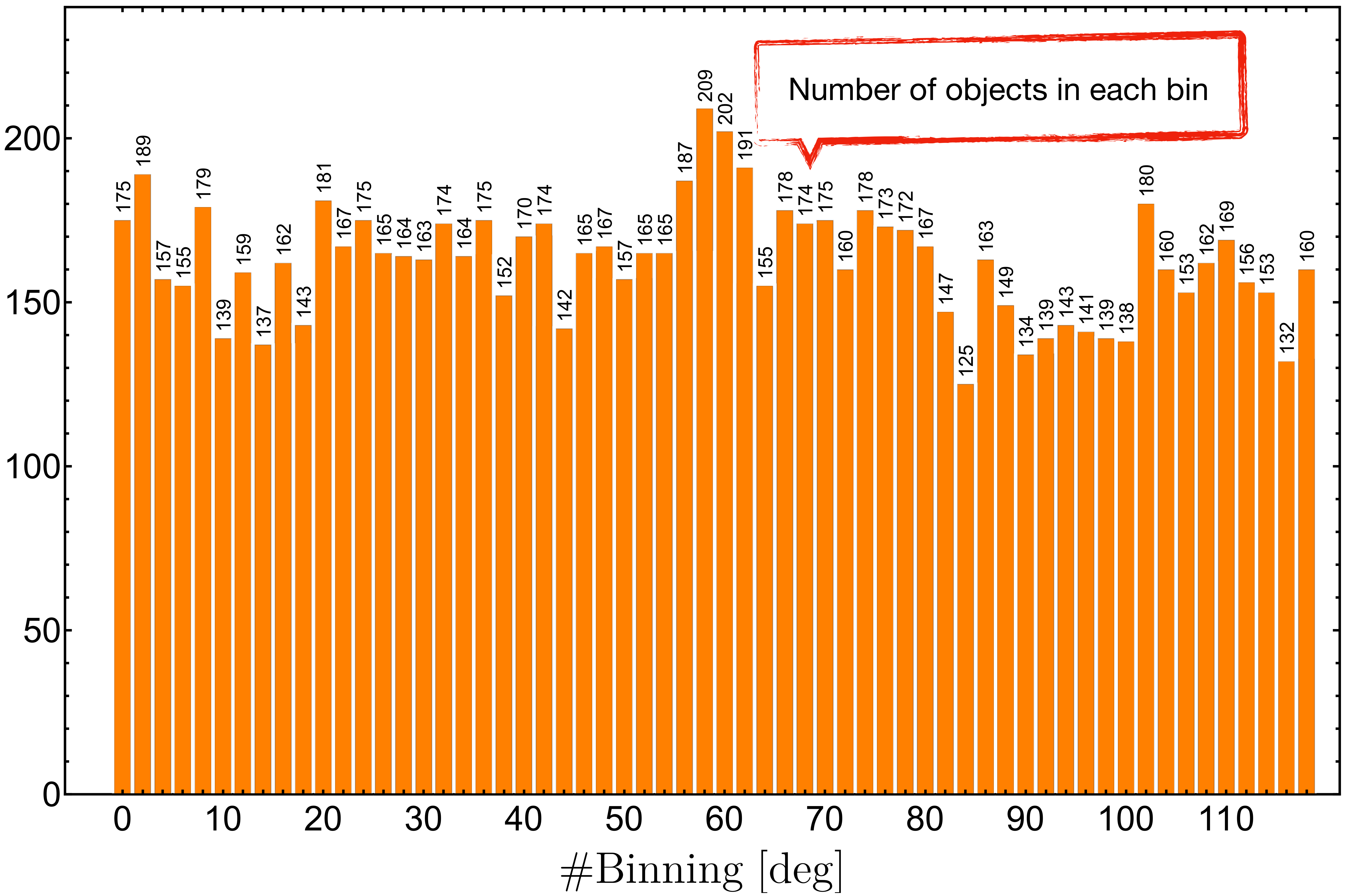
Discretizing into bins



Discretizing into bins

Segmentation algorithms: Generating Bins

Distribution by RANN [Trojans]



Bins rule

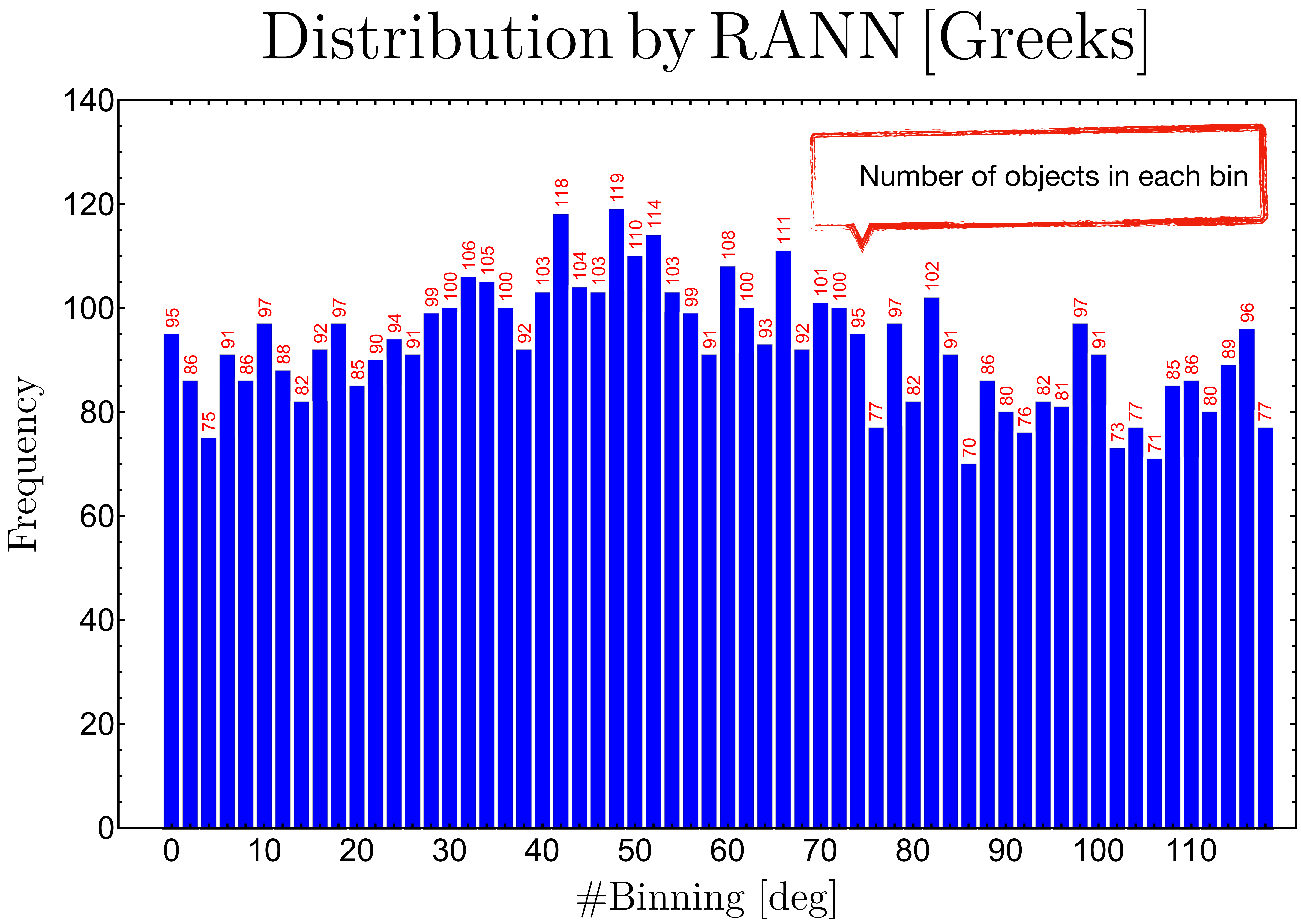
Sturges:

$$B_{St} = k + \log[n].$$

Rice:

$$B_R = 2\sqrt[3]{n}.$$

Segmentation algorithms: Generating Bins



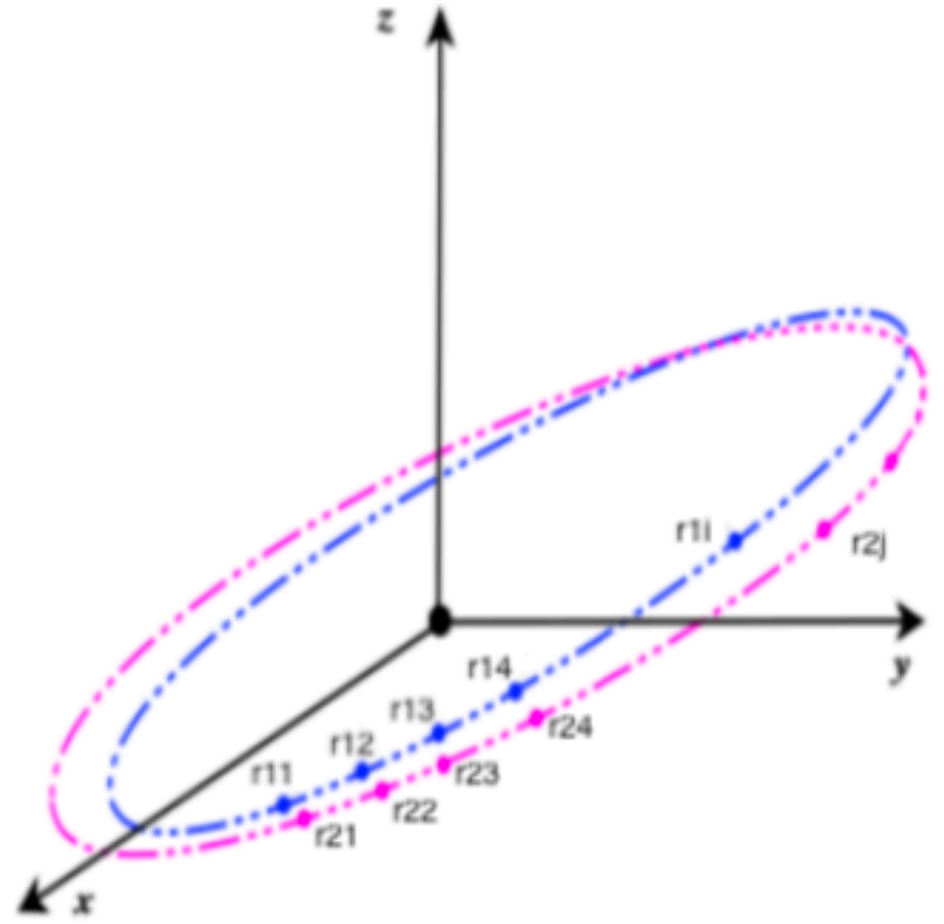
Orbital Pair Discretization Method

Algorithm: maximum of the minimum distances between orbits

1.- Propagate two asteriod objects during one period (OrbitSAT- GME - Unizar)

$$\text{O1: } (t_1, \mathbf{r}_1), (t_2, \mathbf{r}_2), (t_3, \mathbf{r}_3), \ldots, (t_n, \mathbf{r}_n)$$

$$\text{O2: } (t_1, \mathbf{r}_1), (t_2, \mathbf{r}_2), (t_3, \mathbf{r}_3), \ldots, (t_n, \mathbf{r}_n)$$



Orbits

2.- We compute the following minimum distances:

$$\begin{aligned}
m_1 &= \text{mín}\{d(\mathbf{r}_1, \mathbf{r}_1), d(\mathbf{r}_1, \mathbf{r}_2), d(\mathbf{r}_1, \mathbf{r}_3), \ldots, d(\mathbf{r}_1, \mathbf{r}_n)\} = \text{mín}\{d(\mathbf{r}_1, O2)\} \\
m_2 &= \text{mín}\{d(\mathbf{r}_2, \mathbf{r}_1), d(\mathbf{r}_2, \mathbf{r}_2), d(\mathbf{r}_2, \mathbf{r}_3), \ldots, d(\mathbf{r}_2, \mathbf{r}_n)\} = \text{mín}\{d(\mathbf{r}_2, O2)\} \\
m_3 &= \text{mín}\{d(\mathbf{r}_3, \mathbf{r}_1), d(\mathbf{r}_3, \mathbf{r}_2), d(\mathbf{r}_3, \mathbf{r}_3), \ldots, d(\mathbf{r}_3, \mathbf{r}_n)\} = \text{mín}\{d(\mathbf{r}_3, O2)\} \\
\ldots &= \ldots \\
m_n &= \text{mín}\{d(\mathbf{r}_n, \mathbf{r}_1), d(\mathbf{r}_n, \mathbf{r}_2), d(\mathbf{r}_n, \mathbf{r}_3), \ldots, d(\mathbf{r}_n, \mathbf{r}_n)\} = \text{mín}\{d(\mathbf{r}_n, O2)\}
\end{aligned}$$

Minimun distances

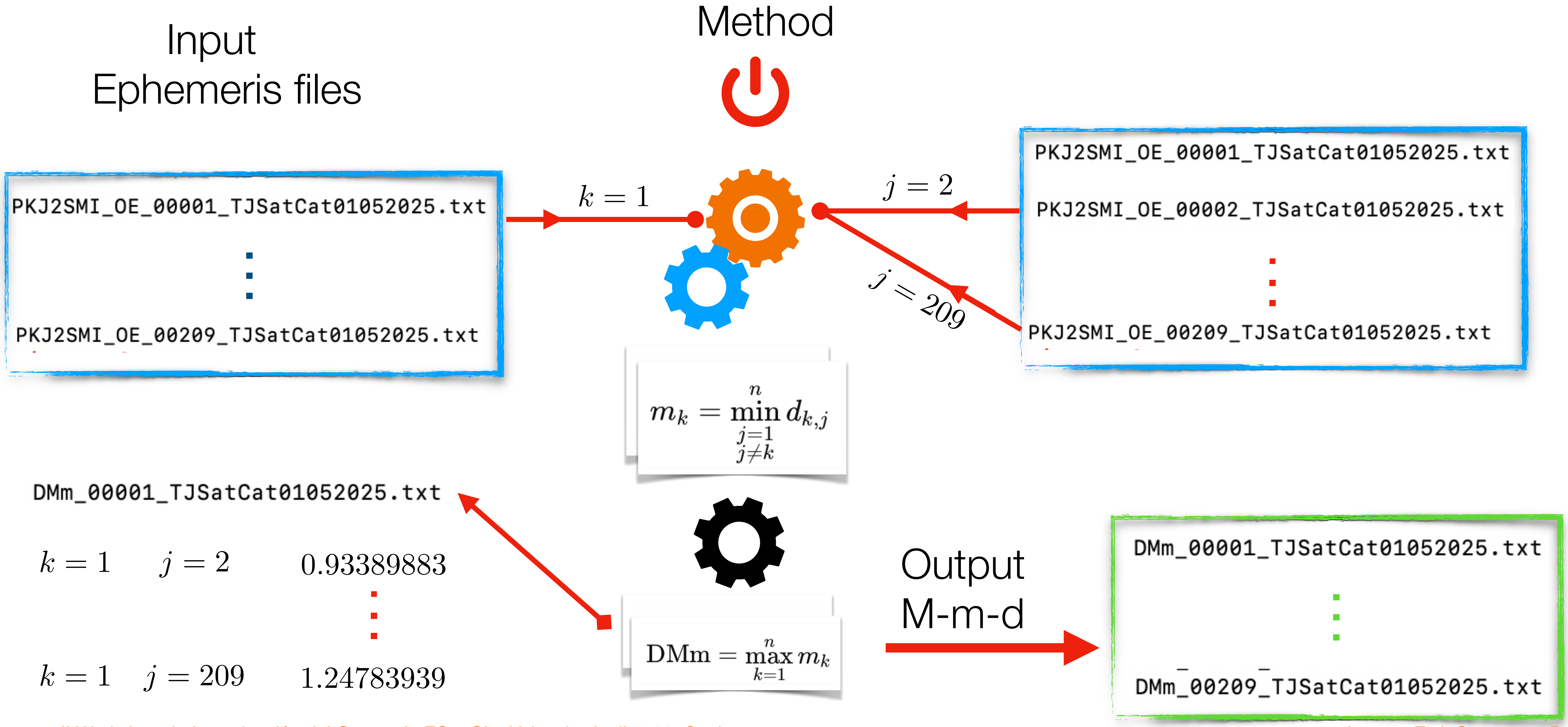
3.- We compute the maximum of the minimums:

$$M = \text{máx}\{m_1, m_2, m_3, \ldots, m_n\}$$

Maximun

Example: Bins #30 for Trojans

Bin # 30. n= 209 objects.



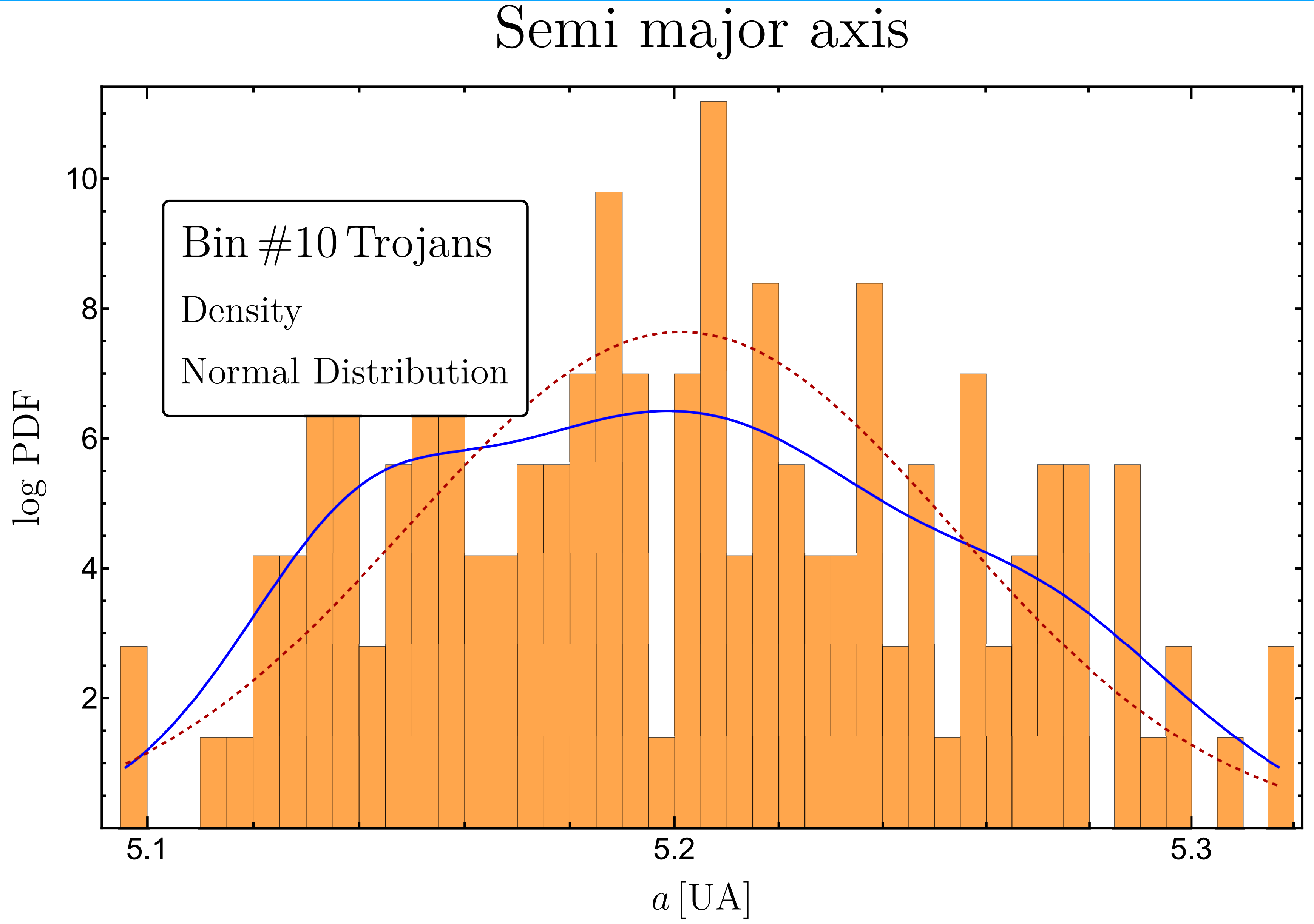
Analysis of the results

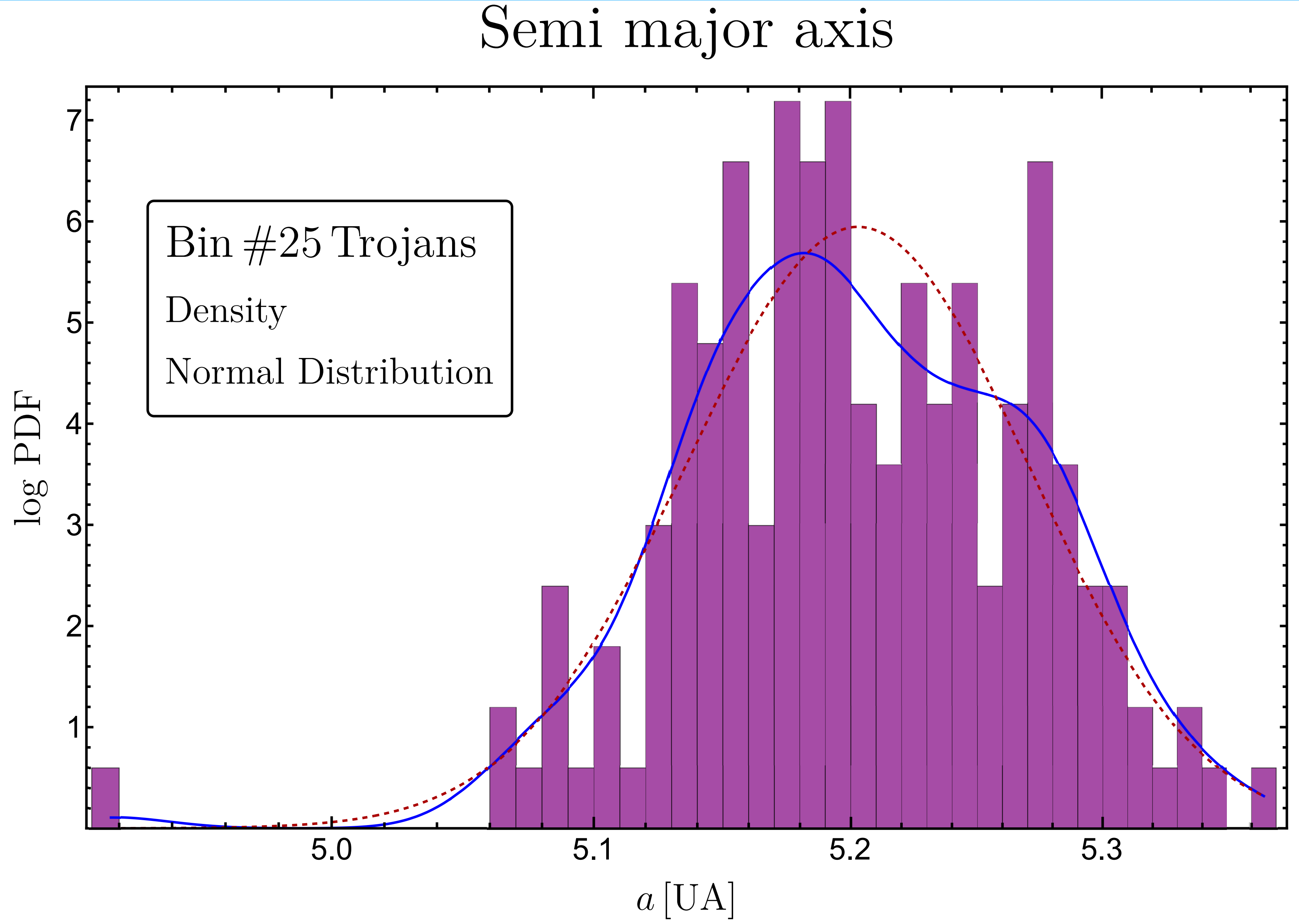
Distribution at the Jupiter

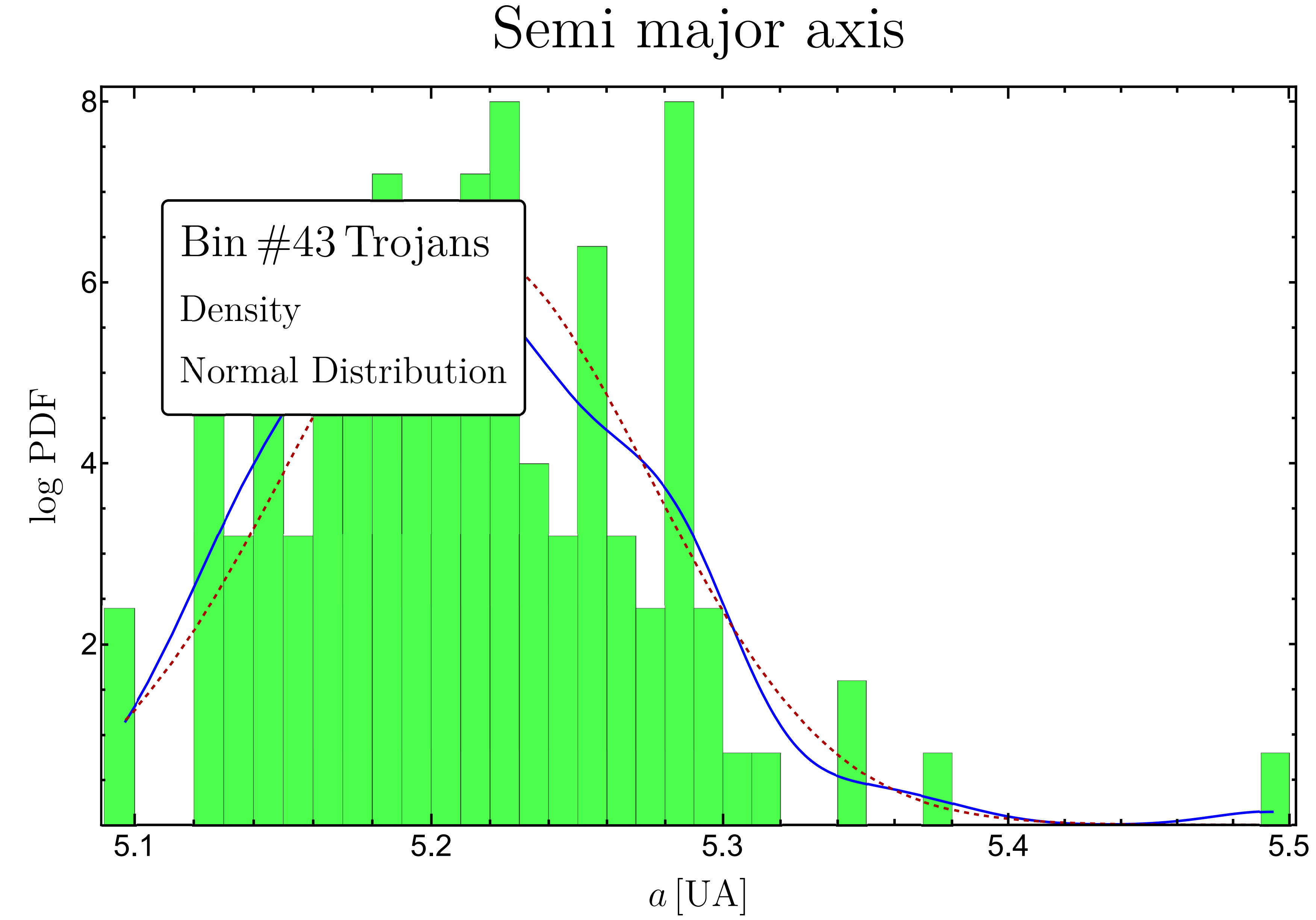
- Execute each bin [Trojans and Greeks]
    - Trojans = 9744 distributed across 60 bins
    - Greeks = 5561 distributed across 54 bins

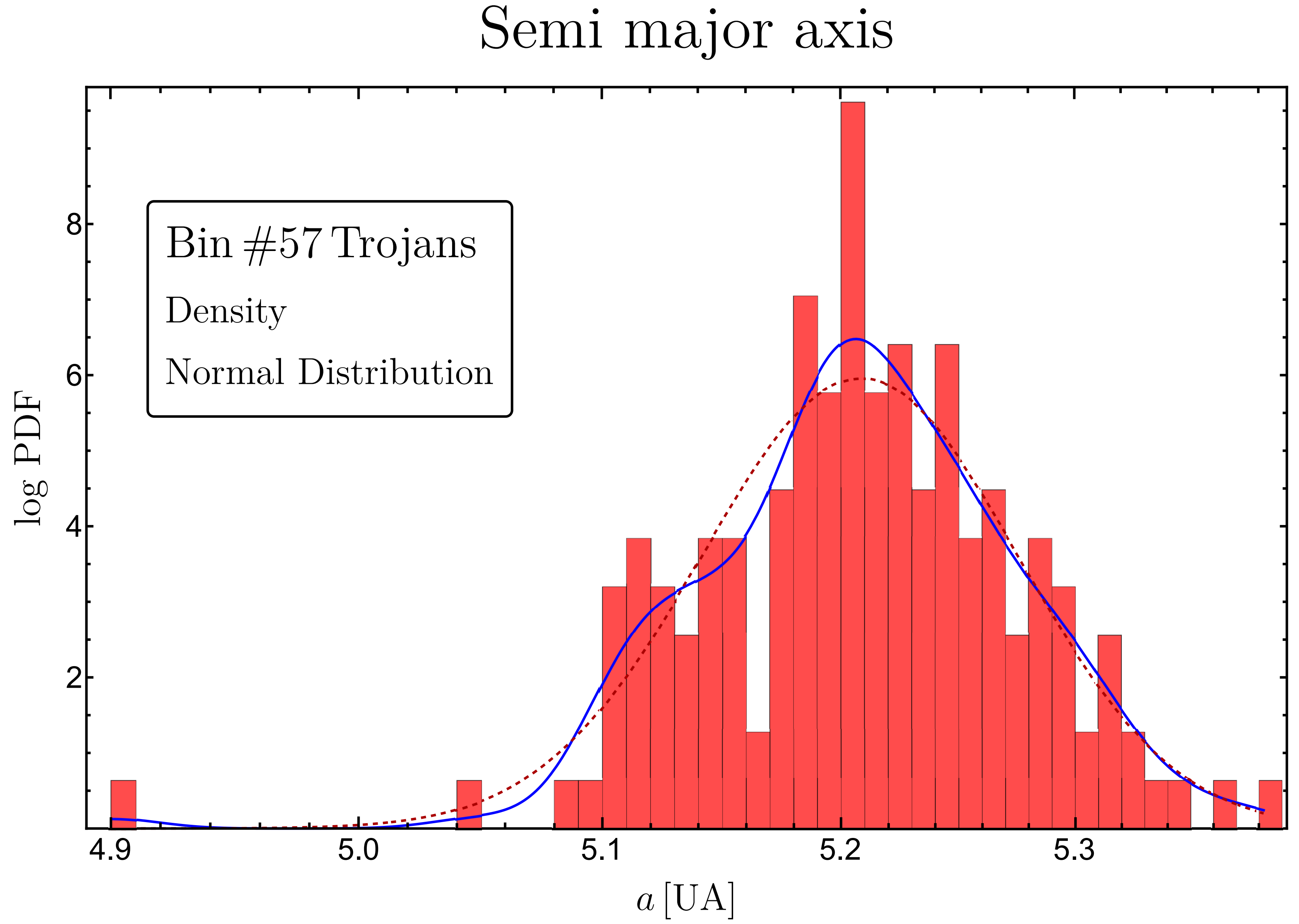
- Threshold distance analysis
    - Trojans = 60 bins [determined 60 distributions and ρ]
    - Greeks = 54 bins [determined 54 distributions and ρ]

Distribution of Semi-major axis by Bin [Trojans]









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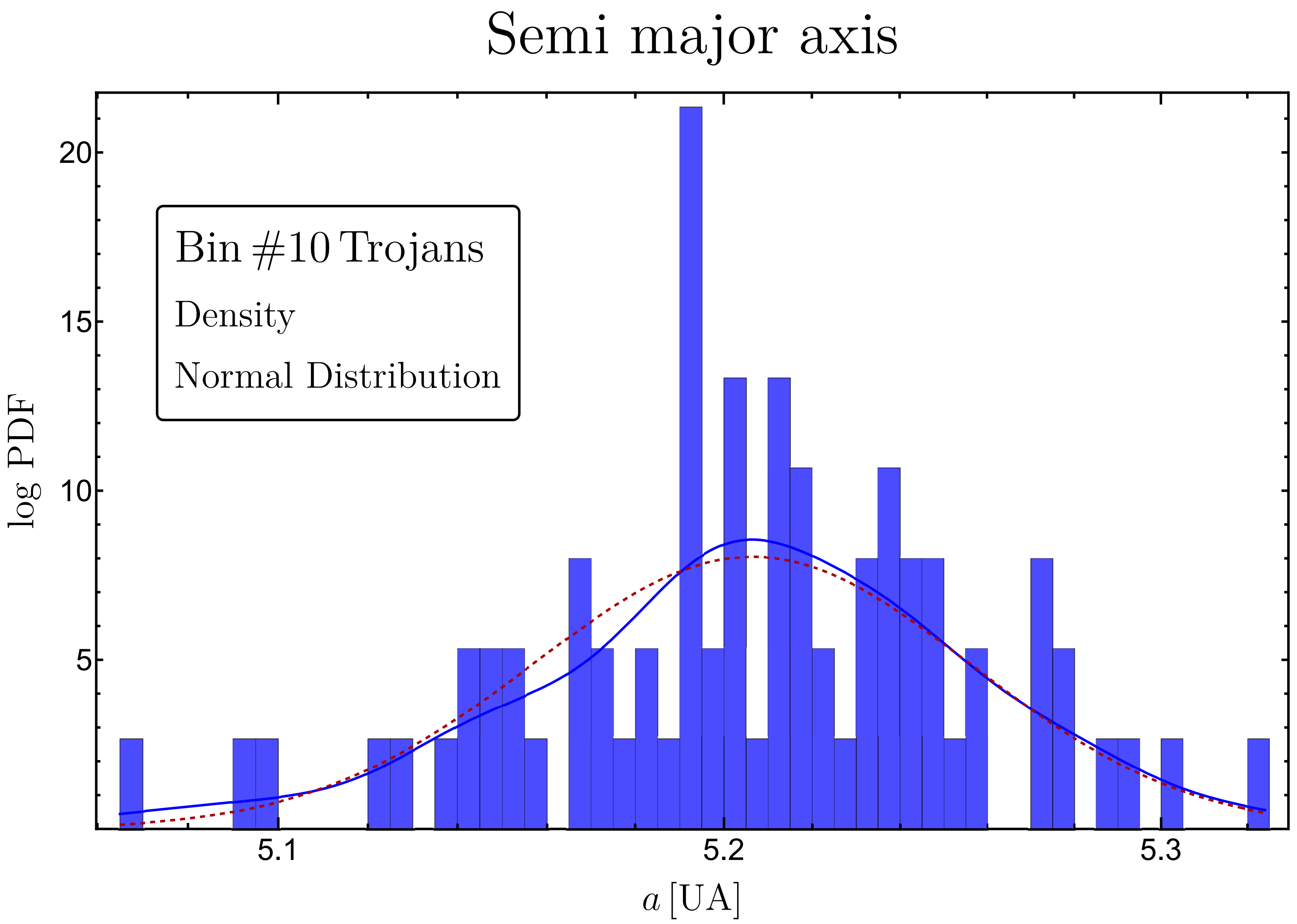
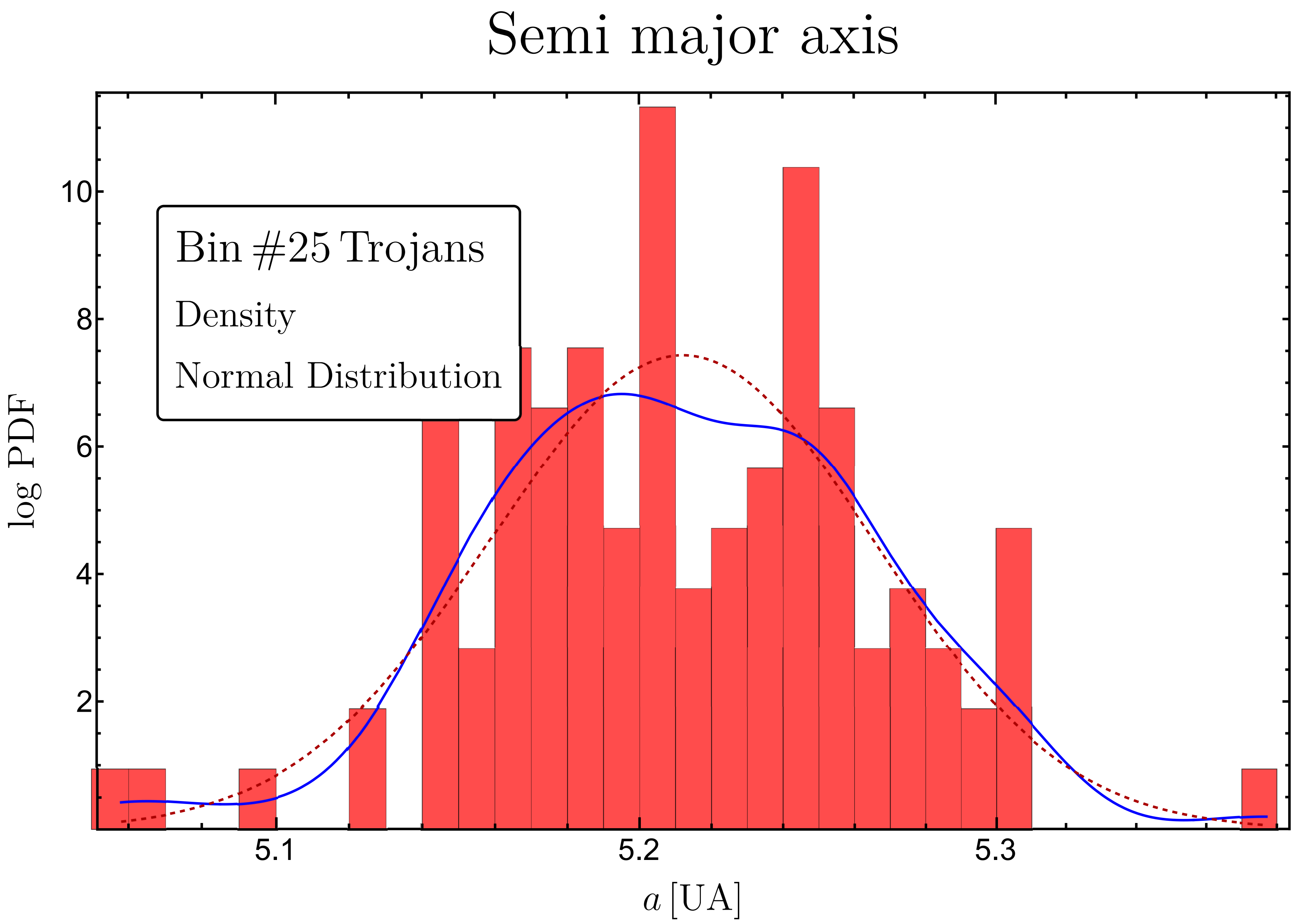
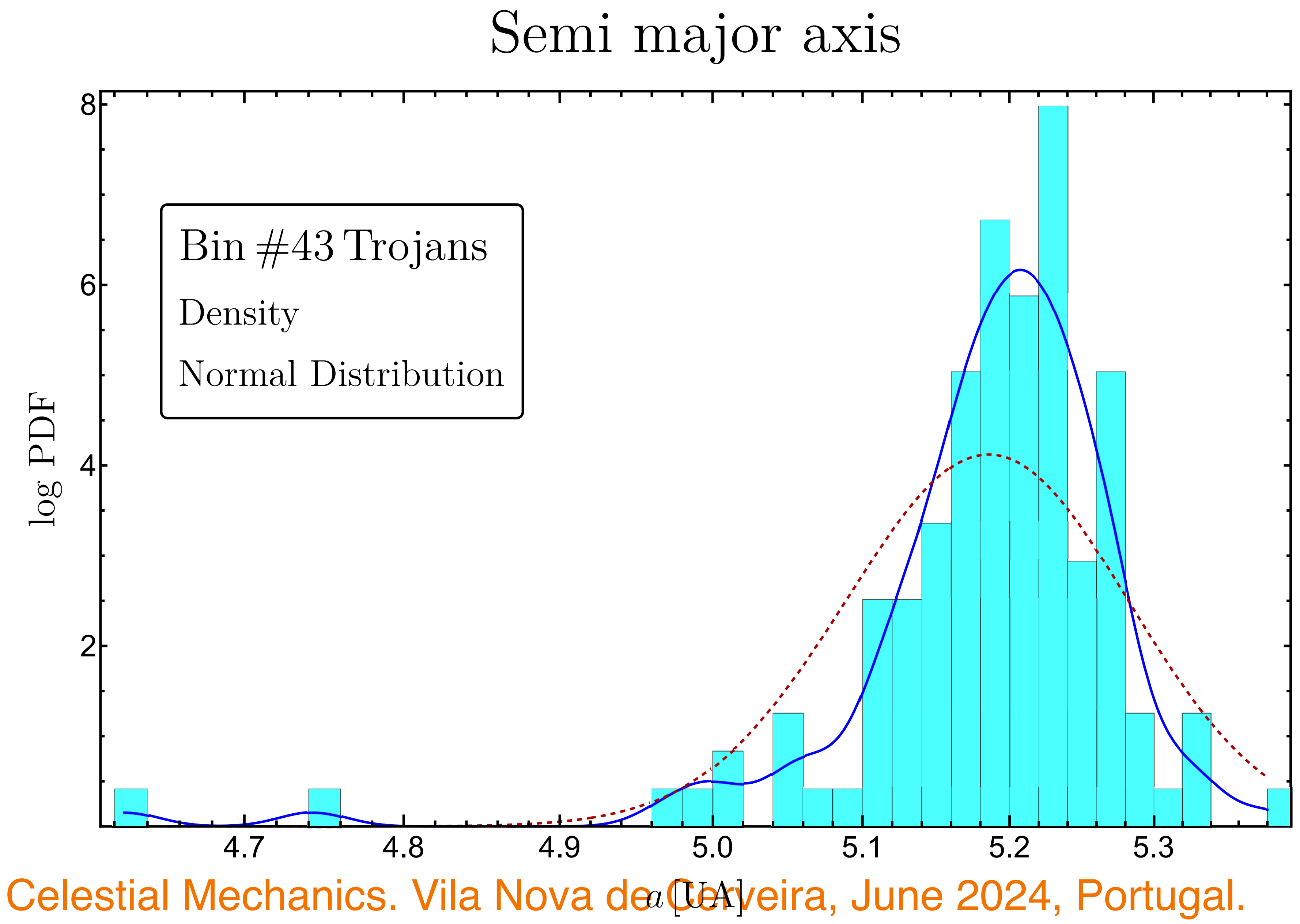
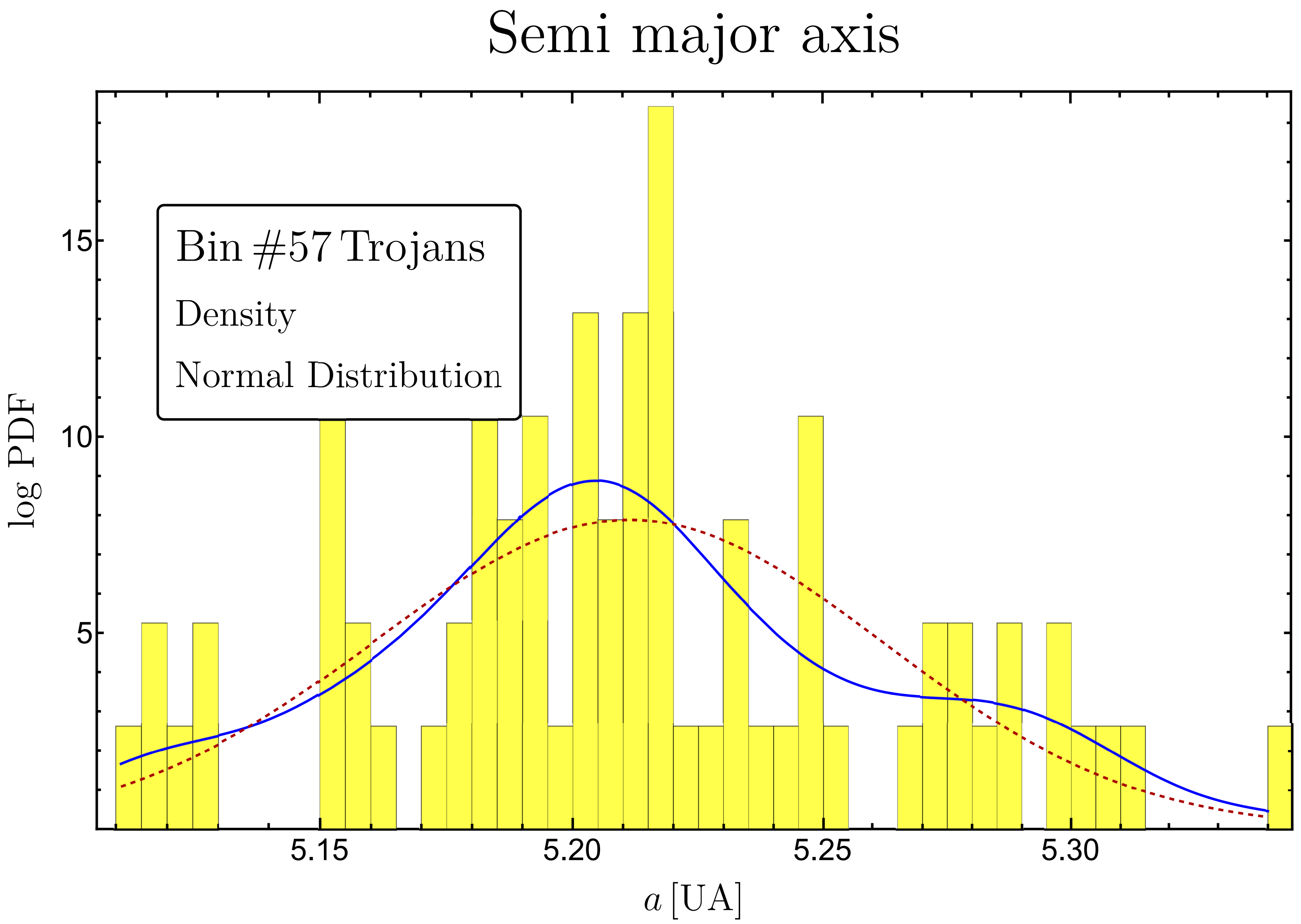
Distribution of Semi-major axis by Bin [Greeks]



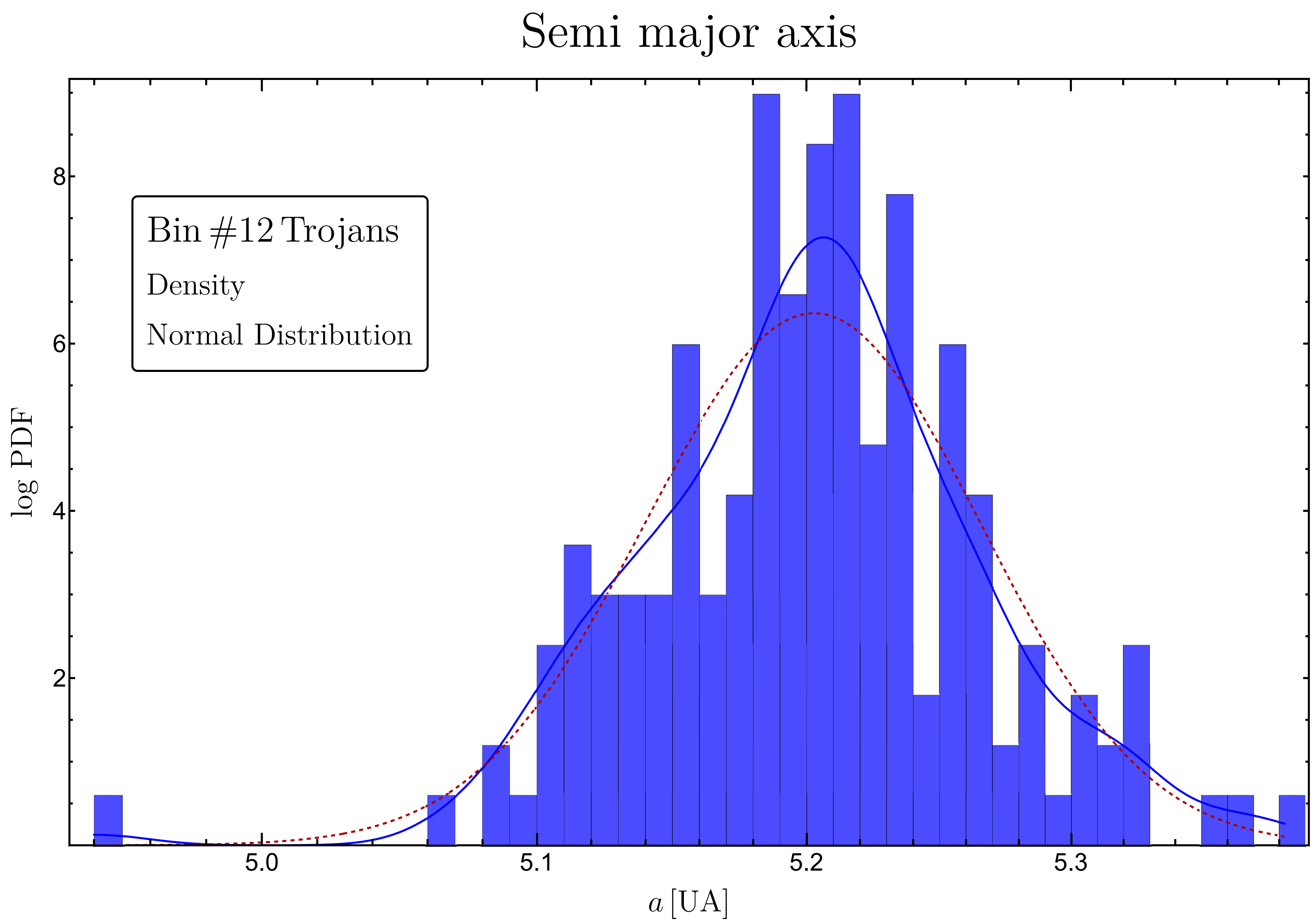






Leadership and Associated Objects (LAO)

Bin #12. n= 167 objects.

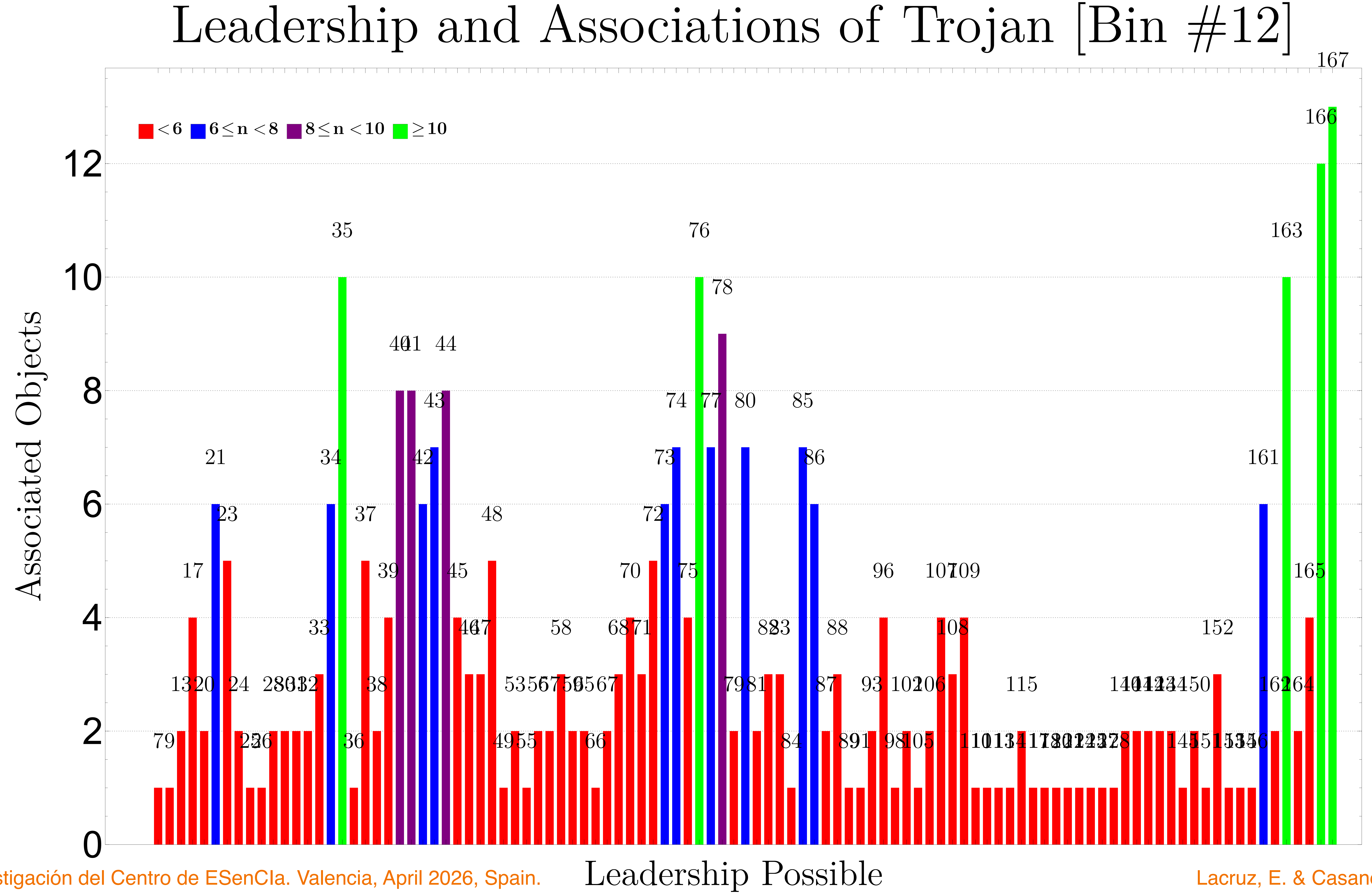


Threshold distance is determined by a *Weibull distribution density.*

$$\mu_{12} = 5.20275\ AU$$

$$\rho_{12} = 2.0\sigma_{12} = 0.5$$

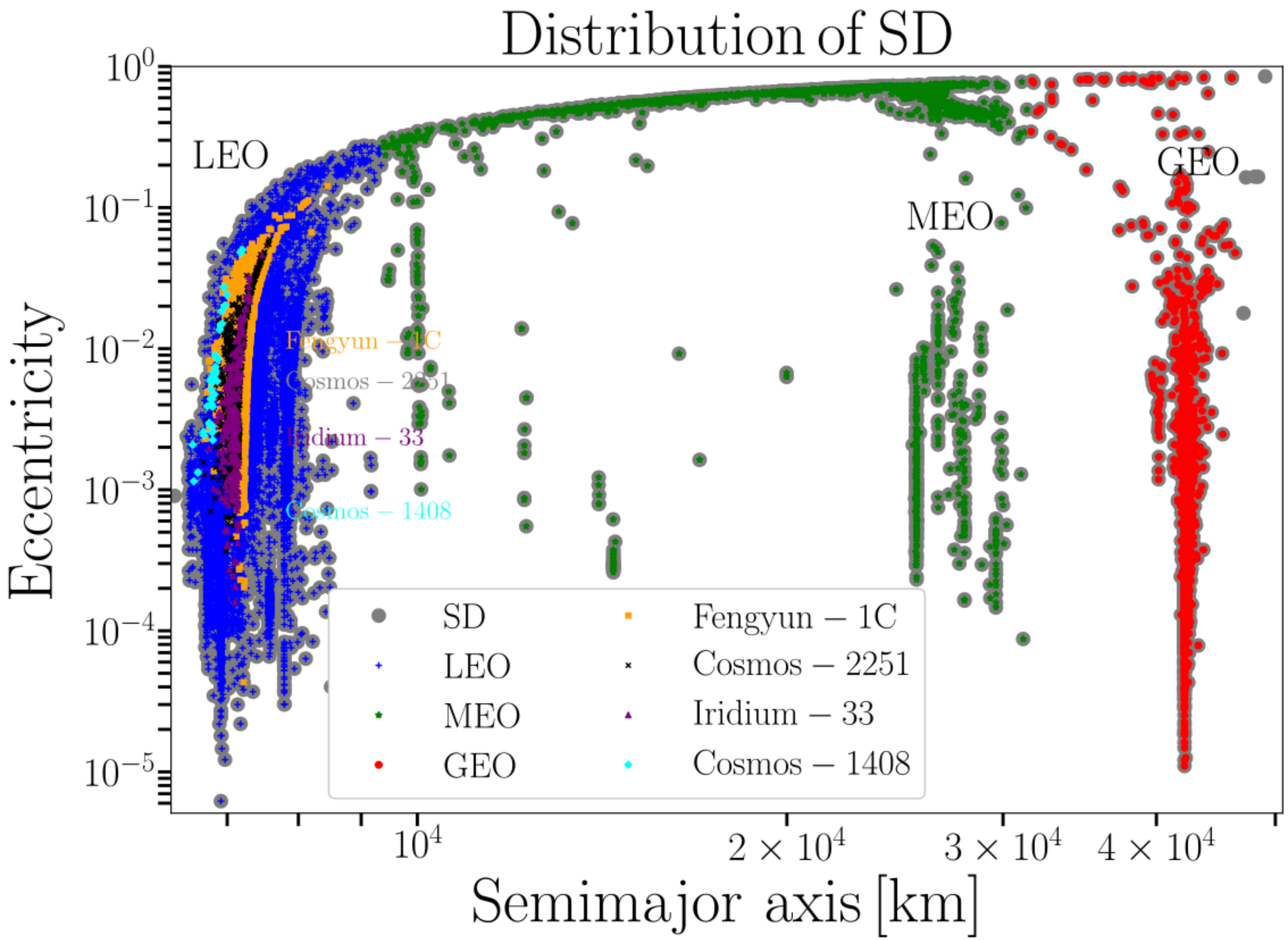
Leadership and Associated Objects (LAO)



Leadership and Associated Objects (LAO)

Leader	2009 UN87	5.313	0.101	13.6	70.8
Associated	2014 EX16	5.193	0.078	3.7	70.8
	2003 GL21	5.233	0.037	9.8	71.2
	2013 CO255	5.210	0.035	8.6	67.0
	2014 FW2	5.221	0.121	12.6	70.2
	2010 UK97	5.173	0.140	13.9	69.7
	2020 QY92	5.207	0.139	8.8	67.9
	2020 RU86	5.323	0.062	14.4	68.6
	2021 TO75	5.282	0.085	8.5	67.4
	2021 TN155	5.242	0.081	8.3	69.4
	2021 US120	5.186	0.114	9.0	66.4
	2021 VN17	5.135	0.094	8.7	71.7
No associated	2009 SV361	5.210	0.152	14.4	265.3
	2009 TZ24	5.199	0.079	9.6	45.9

Thanks



Distribution of SD
Eccentricity
Semimajor axis [km]
LEO
MEO
GEO
Fengyun – 1C
Cosmos – 2251
Iridium – 33
Cosmos – 1408
SD
LEO
MEO
GEO
Fengyun – 1C
Cosmos – 2251
Iridium – 33
Cosmos – 1408